# Secure and practical Quantum Digital Signatures

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Digital signatures represent a crucial cryptographic asset that must be protected against quantum adversaries. Quantum Digital Signatures (QDS) can offer solutions that are information-theoretically (IT) secure and thus immune to quantum attacks. In this work, we analyze three existing practical QDS protocols based on preshared secure keys (e.g., established with quantum key distribution) and universal hashing families. For each protocol, we make amendments to close potential loopholes and prove their IT security while accounting for the failure of IT-secure authenticated communication. We then numerically optimize the protocol parameters to improve efficiency in terms of preshared bit consumption and signature length, allowing us to identify the most efficient protocol.

## I. INTRODUCTION

Popular public-key digital signature (DS) protocols, such as the digital signature algorithm (DSA) [1] and the elliptic curves DSA [2], can be broken by large-scale fault-tolerant quantum computers. The new post-quantum DS standards FIPS 204 [3] and FIPS 205 [4] are not (yet) efficiently attackable by quantum computers, but their security is still based on computational assumptions. Conversely, the appeal of quantum digital signatures (QDS) lies in their information-theoretic (IT) security, i.e., security against computationally unbounded adversaries, analogously to quantum key distribution (QKD).

DSs based on a public-key infrastructure can be verified by any user by simply retrieving the public key of the sender, which requires a trusted third party (a certificate authority) certifying that the public key actually belongs to the sender. Conversely, QDS schemes aiming at IT security do not rely on a trusted authority and may require pairwise preshared keys between all users, which can limit the number of users able to verify a signature. For this reason, QDS schemes have been mostly analyzed in the simplest non-trivial scenario, i.e. a tripartite scenario with a sender, a receiver and an additional verifier.

The first QDS was proposed in 2001 by Gottesman and Chuang [5], with challenging experimental requirements such as quantum memories, secure quantum channels and swap tests. Later protocols removed the requirement of quantum memories but were still based on authenticated – hence secure<sup>1</sup>– quantum channels [6]. In 2016, more practical QDS schemes were proposed without the need of secure quantum channels [7, 8]. However, the scheme in Ref. [7] is not proved secure under the more general type of attacks. Anyways, the main drawback of previous QDS schemes, including Refs. [7, 8], is that they can only sign single-bit messages, thus resulting inefficient for long messages.

To overcome the security and efficiency limitations of previous QDS proposals, three promising QDS schemes have been proposed, respectively by Yin et al. in Ref. [9], by García Cid et al. in Ref. [10] and by Amiri et al. in Ref. [11]. These schemes can sign arbitrarily long documents with relatively short signatures, thereby improving on the efficiency of previous proposals by several orders of magnitude. The quantum nature of these schemes lies in leveraging preshared QKD keys (and universal hashing families) to achieve IT security –except for Ref. [10], due to the use of computationally secure hash functions.

Recently, the protocol by Yin et al. was further developed to remove the need for privacy amplification [12] and also error correction [13] in the QKD protocols preceding the signature scheme. In parallel, the protocol by García Cid et al. was improved in Ref. [14] in terms of security bounds and signature length, though still lacking the IT security of the other two protocols. The protocol by Amiri et al. was modified in Ref. [15], where the authors improve the scalability of the protocol for large networks of receivers by introducing a smaller and relatively trusted subnetwork that is tasked with verifying the signatures on the proxy of all other nodes. In the process, the authors also close a security loophole of the original protocol [11] and prevent dishonest coalitions of receivers from enforcing non-transferability by limiting the number of dishonest users compared to Ref. [11].

On the experiments side, Refs. [16] and [17] report the first experimental realization of a QDS protocol with discrete and continuous variable encoding, respectively. More recent experiments [9, 18, 19] implemented efficient QDS schemes [11, 12] in realistic quantum networks, demonstrating multiple contract signatures within one second [19].

In this work, we perform an in-depth analysis of the three seminal protocols from Refs. [9–11], by paying particular attention to the use of authenticated channels. This entails, for each protocol, reviewing which communication steps require authentication in order to avoid security loopholes, followed by proving their IT security while taking into account the failure of IT-secure authenticated channels, unlike previous works [9–15]. In the process, we also tighten the security bounds compared

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Note that an authenticated quantum channel must be also a private channel. Indeed, any attempt to learn about the quantum state traveling in the channel leads to a modification of the state, which implies the channel is no longer authentic. Hence, authenticity implies privacy for quantum channels.

to those derived in Refs. [9–11], where possible. In the case of the protocol by García Cid et al. [10], we propose a modified protocol which achieves IT security by replacing computationally secure hashes with  $\varepsilon$ -almost XOR universal<sub>2</sub> hash families [20]. For the protocol by Amiri et al. [11], we address the security loophole raised in Ref. [15] with a complementary approach to that of Ref. [15].

After having established the IT security of the three protocols from Refs. [9–11] on an equal footing, we compare their performance in a tripartite scenario of one sender and two receivers. In particular, we numerically optimize their parameters when signing documents of various sizes at a given security threshold and deduce which protocol is more efficient in terms of consumed preshared bits and length of the signature. In doing so, we account for the preshared bits needed to agree on universal hashing functions, to establish secret communication with one-time pad encryption, and to establish the IT-secure authenticated channels.

The paper is organized as follows. In Sec. II we describe the adversarial scenario and summarize the universal families adopted by the QDS protocols. In Sec. III we detail the analyzed QDS protocols and prove their security. In Sec. IV we benchmark the performance of the protocols. We provide concluding remarks in Sec. V. Appendix A contains a proof of IT-secure message authentication while Appendix B details the universal families used in the manuscript. Appendix C contains the security proofs of the QDS protocol based on Ref. [11].

# II. BACKGROUND

In order to analyze the three QDS protocols from Refs. [9–11] in the tripartite scenario, we adopt a uniform notation across the protocols to improve readability.

The document to be signed is indicated as Doc and the corresponding signature is Sig. Together, they form the document-signature pair  $\{Doc, Sig\}$ . The bit length of Doc is  $b_M$ .

In our tripartite scenario, Alice is the sender and produces the pair  $\{Doc, Sig\}$ , Bob is the receiver of the pair  $\{Doc, Sig\}$  and Charlie is the honest the verifier. Alice and Bob can be malicious, but not both at the same time. Specifically, a malicious Alice can perform a repudiation attack, in which she wants to convince Charlie that the pair  $\{Doc, Sig\}$  verified by Bob is not authentic. Alternatively, a malicious Bob can perform a forgery attack where he produces a forged pair  $\{Doc', Sig'\}$  and wants Charlie to accept the forged pair. We prove the security of the analyzed QDS protocols against forgery and repudiation attacks, according to the following definitions.

**Definition II.1.** A QDS protocol is  $\varepsilon_{for}$ -secure against forgery attacks if the probability that the protocol does not abort and that Charlie accepts a pair  $\{Doc', Sig'\}$  forged by Bob is at most  $\varepsilon_{for}$ .

**Definition II.2.** A QDS protocol is  $\varepsilon_{\text{rep}}$ -secure against repudiation attacks if the probability that the protocol does not abort and that Charlie rejects a pair  $\{Doc, Sig\}$  accepted by Bob is at most  $\varepsilon_{\text{rep}}$ .

The analyzed QDS protocols [9–11] achieve IT security with respect to forgery and repudiation attacks by extending IT-secure message authentication codes – specifically, Wegman-Carter (WC) authentication based on universal hashing [21]– to scenarios with multiple receivers, while making sure that no receiver can pretend to be the sender. In the considered QDS protocols, Alice generates the signature by hashing the  $b_M$ -bit document with hash functions derived from either an  $\varepsilon$ -almost XOR universal<sub>2</sub> family or an  $\varepsilon$ -almost strongly universal<sub>2</sub> family, generating hashes of  $b_H$  bits each. We define both families for completeness below.

**Definition II.3.** Let  $\mathcal{F}_{ASU} = \{f : M \to B\}$  be a family of hash functions. Then,  $\mathcal{F}_{ASU}$  is  $\varepsilon$ -almost strongly universal<sub>2</sub> ( $\varepsilon$ -ASU<sub>2</sub>) if:

$$\forall m_1 \neq m_2, \forall b_1, b_2 \in B$$

$$\Pr_{f \in R\mathcal{F}_{ASU}} [f(m_1) = b_1 \land f(m_2) = b_2] \leq \frac{\varepsilon}{2^{b_H}}, \quad (1)$$

where  $f \in_R \mathcal{F}$  indicates that the function f is sampled randomly from the set  $\mathcal{F}$  and where  $b_H$  is the bit length of the elements in B.

**Definition II.4.** Let  $\mathcal{F}_{AXU} = \{f : M \to B\}$  be a family of hash functions. Then,  $\mathcal{F}_{AXU}$  is  $\varepsilon$ -almost XOR universal<sub>2</sub> ( $\varepsilon$ -AXU<sub>2</sub>) if:

$$\forall m_1 \neq m_2, \forall b \quad \Pr_{f \in_R \mathcal{F}_{AXU}} [f(m_1) \oplus f(m_2) = b] \leq \varepsilon.$$
 (2)

Note that  $\varepsilon$ -ASU<sub>2</sub> families are a subset of  $\varepsilon$ -AXU<sub>2</sub> families since (1) implies (2).

In Table I we report the two specific families adopted by the analyzed QDS protocols, summarizing their main characteristics.

Table I. The two universal hashing families considered in this manuscript, mapping messages of  $b_M$  bits to tags of  $b_H$  bits. We report the number of preshared key bits consumed to agree on a specific function of the family and the security parameter. More details are found in Appendix B.

Family	Preshared bits	$\varepsilon ext{-security}$
$\mathcal{F}_{ ext{ASU}}$	$3b_H + 2\log_2\left(\frac{b_M}{b_H} - 1\right)$	$2^{1-b_H}$
$\mathcal{F}_{\mathrm{AXU}}$	$2b_H$	$b_M 2^{1-b_H}$

Both universal families can be used in a WC authentication scheme to attain IT-secure message authentication. In brief, suppose that a sender and a receiver share a random secret string that uniquely identifies a hash function f from  $\mathcal{F}_{ASU}$ . Then, the sender wishing to send

a message m sends the tuple (m,t) to the receiver over a public channel, where the tag is t = f(m). Let (m',t') be the tuple received the by the receiver. The receiver accepts the message as authentic if f(m') = t'. By the property (1) of  $\varepsilon$ -ASU<sub>2</sub> families, we deduce that an attacker trying to forge a pair (m',t') with  $m \neq m'$  that is accepted by Bob only succeeds with probability  $\varepsilon$  [22].

An analogous result can be proved for  $f \in \mathcal{F}_{AXU}$ . Indeed, if the sender and receiver additionally share a random string r and the tag is computed as  $t = f(m) \oplus r$  (the symbol  $\oplus$  indicates addition modulo 2), the property (2) of  $\varepsilon$ -AXU<sub>2</sub> families ensures that an attacker forging a new pair only succeeds with probability  $\varepsilon$  [20].

Interestingly, as first proposed by WC [21], it is possible to authenticate n messages using the same hash function from either family,  $\mathcal{F}_{AXU}$  or  $\mathcal{F}_{ASU}$ , by OTP-encrypting the tags with fresh random strings, such that the attack probability on any of the n messages is still  $\varepsilon$  [22]. This procedure is sometimes called key recycling and we provide a proof of its security when employing the  $\varepsilon$ -ASU<sub>2</sub> family in Appendix A.

In each QDS protocol, it is implicitly assumed that if a message sent over the authenticated channel is not successfully authenticated, then the protocol aborts (which is not equivalent to rejecting the signature). In order to compare the QDS protocols on an equal footing while accounting for the failure of IT-secure authenticated channels, we assume that all authenticated channels implement WC authentication with key recycling. Specifically, we employ functions from the  $\mathcal{F}_{\rm AXU}$  family (see Table I), producing hashes of  $b'_H$  bits that are protected by OTP with fresh secret bits. Therefore, the number of preshared bits consumed by sending n messages over an authenticated channel is  $(2+n)b'_H$ , where we also accounted for the bits required to agree on a specific element from  $\mathcal{F}_{\rm AXU}$ .

## III. QDS PROTOCOLS

In this section we analyze three practical QDS protocols based on the protocols introduced in Refs. [9–11]. For each protocol, we provide a detailed description highlighting the differences with respect to its original formulation and a security proof against forgery and repudiation attacks as defined in Sec. II. Our security proofs account for the failure probability of WC authentication, unlike their original derivations, while all preshared secret bits are assumed to be perfectly secure.

## A. QDS by Yin et al.

The QDS scheme introduced in Ref. [9] resembles a tripartite classical DS scheme, where the security is lifted to IT security by replacing public-key cryptography with the use of one-time pad (OTP) and one-time universal<sub>2</sub>

hashing (OTUH), i.e., hash functions from the  $\mathcal{F}_{AXU}$  family that are renewed for each document signature.

#### 1. The protocol

We modify the original formulation of the protocol from [9] such that Charlie always verifies the signature after receiving the outcome of the verification of Bob, regardless of Bob accepting it or not. In this way, Charlie can detect if Bob is a liar who accepts falsified signatures or rejects original signatures. We believe this is an additional useful feature of the protocol.

# Protocol 1 QDS protocol [9]

- 1. Distribution stage Alice, Bob and Charlie run a QKD or quantum secret sharing protocol, in order to establish secret keys  $X_A$  for Alice,  $X_B$  for Bob, and  $X_C$  for Charlie, with the property that  $X_A = X_B \oplus X_C$ . We partition each of the keys into two partitions. The first  $b_H$  bits of each key is labeled as  $X_A^{b_H}$ ,  $X_B^{b_H}$ , and  $X_C^{b_H}$ , respectively, while the following  $2b_H$  bits are labeled as  $X_A^{2b_H}$ ,  $X_B^{2b_H}$ , and  $X_C^{2b_H}$ . It holds:  $X_A = X_A^{b_H} \cup X_A^{2b_H}$  and similarly for  $X_B$  and  $X_C$ .
- 2. Signing of Alice Alice generates a  $b_H$ -bit random string,  $p_a$ , with elements  $(p_a)_i$ . After associating to  $p_a$ the following polynomial over GF(2):  $p_a(x) = x^{b_H} + (p_a)_{b_H-1}x^{b_H-1} + \cdots + (p_a)_1x + (p_a)_0$  of degree  $b_H$ , Alice checks whether  $p_a(x)$  is irreducible (see algorithm in Supplementary Material of [9]). If the test is negative, Alice generates a new random string until the corresponding polynomial is irreducible. From the irreducible polynomial  $p_a(x)$  and the key  $X_A^{bH}$ , Alice defines a linear feedback shift register (LFSR) and obtains the associated To eplitz matrix  $T_{p_a,X_A^{b_H}},$  which is an element of  $\mathcal{F}_{\mathrm{AXU}}$ (see Appendix B). Then, she computes the  $b_H$ -bit hash value of the document:  $h_a = T_{p_a, X_A^{b_H}} \cdot Doc$  and the digest:  $Dig = (h_a||p_a)$  as the concatenation of the hash of the document with the random string  $p_a$ . Finally, Alice derives the signature by encrypting the digest with the secret key  $X_A^{2b_H}$  via OTP:  $Sig = Dig \oplus X_A^{2b_H}$ . The couple  $\{Doc, Sig\}$  is sent to the receiver, Bob, over a public channel.
- 3. Verification of Bob Firstly, Bob sends via an authenticated channel the received couple  $\{Doc, Sig\}$  and the key  $X_B$  to Charlie. Once Charlie receives the data from Bob, he sends  $X_C$  to Bob over the same authenticated channel. Bob uses the key from Charlie to recover Alice's key, by computing:  $K_B = X_B \oplus X_C$ . Using  $K_B^{2bH}$  Bob recovers the digest by computing  $K_B^{2bH} \oplus Sig$  and retrieves, in particular, the bit string  $p_b$  and the hash  $h_b$  produced by Alice. Bob generates the LFSR-based Toepliz matrix using  $p_b$  and the string  $K_B^{bH}$ ,  $T_{p_b,K_B^{bH}}$ , and computes the hash  $h_b' = T_{p_b,K_B^{bH}} \cdot Doc$ . Bob accepts the signature if  $h_b = h_b'$  and informs Charlie by sending him the bit

 $V_B = 0$  over the authenticated channel. Otherwise, if  $h_b \neq h'_b$ , Bob rejects the signature and sends  $V_B = 1$  to Charlie.

4. Verification of Charlie Charlie uses his key and the key received from Bob to compute  $K_C = X_C \oplus X_B$ . Then, he employs  $K_C^{2b_H}$  and the Sig from Bob to acquire an expected digest:  $K_C^{2b_H} \oplus Sig$ , i.e. the bit string  $p_c$  and the hash  $h_c$  produced by Alice. Subsequently, Charlie generates the LFSR-based Toeplitz matrix  $T_{p_c,K_C^{b_H}}$  and computes the hash  $h'_c = T_{p_c,K_C^{b_H}} \cdot Doc$ . Charlie accepts the signature if  $h_c = h'_c$ , otherwise he rejects it.

### 2. Security proof

In the following, we rederive the security of Protocol 1 with respect to forgery and repudiation. In doing so, we account for the potential failure of the authenticated channel between Bob and Charlie, which was not considered in the original paper [9]. Moreover, we show that failing to renew the hash function from  $\mathcal{F}_{AXU}$  for each document signature opens a security loophole.

**Lemma III.1.** Protocol 1 is  $\varepsilon_{\text{for}}$ -secure against forgery according to Definition II.1, with  $\varepsilon_{\text{for}} = b_M/2^{b_H-1}$ .

*Proof.* There are two cases of forgery attacks: 1) Alice does not sign any document and Bob forges a new pair  $\{Doc', Sig'\}$ ; 2) Alice sends  $\{Doc, Sig\}$  to Bob and Bob forges a modified pair  $\{Doc', Sig'\}$ .

In the first case, Bob has no information from Alice and in particular no information on Alice's key  $X_A^{2b_H}$ . Suppose the worst-case scenario where Bob forwards to Charlie a document Doc' and a digest  $Dig' = (h'_a, p'_a)$  that is consistent with the document:  $h'_a = T_{p'_a, X_A^{b_H}} \cdot Doc'$ . Charlie will recover the digest Dig' and hence validate the signature if and only if Bob correctly guesses the encryption key  $X_A^{2b_H}$  used to encrypt the digest. Therefore, the attack will succeed with probability  $2^{-2b_H}$ .

In the second case, Bob's goal is to find a message m and a hash t such that:  $t = T_{p_a, X_A^{b_H}} \cdot m$ . Indeed, if he succeeds, he sends  $\{Doc', Sig'\}$  with  $Doc' = Doc \oplus m$  and  $Sig' = Sig \oplus (t||0)$ . This pair will be validated by Charlie, since the expected hash by Charlie is:  $h_c = T_{p_a, X_A^{b_H}} \cdot (Doc \oplus m) = h_a \oplus t$ , which coincides with the hash sent by Bob in Sig'. Therefore, the goal of Bob coincides with the goal of an adversary in WC authentication implemented with the family  $\mathcal{F}_{\rm AXU}$ , where the tags are protected by OTP. From Table I, we have that Bob's attack succeeds at most with probability  $b_M/2^{b_H-1}$ .

In conclusion, by putting together the two cases, Bob's forgery attacks succeed with probability at most  $\varepsilon_{\text{for}} = b_M/2^{b_H-1}$ , which concludes the proof.

**Lemma III.2.** Protocol 1, with  $b_M > b_H$ , is  $\varepsilon_{\text{rep}}$ -secure against repudiation according to Definition II.2, with  $\varepsilon_{\text{rep}} = (2b_H + b_M)/2^{b_H'-1}$ .

Proof. Since Bob and Charlie communicate via an authenticated channel and behave honestly, Charlie obtains the same pair  $\{Doc, Sig\}$  received by Bob and the key  $X_B$ . Bob in turn obtains  $X_C$ . Thus, both Bob and Charlie recover Alice's key:  $K_B = K_C = X_A$ . Using  $X_A$  and the pair  $\{Doc, Sig\}$ , both Bob and Charlie will reach the same conclusion about Alice's signature. Therefore, it is impossible that they disagree, unless Alice successfully tampers with the authenticated channel.

Given the implementation of the authenticated channel described in Sec. II, the probability that Alice successfully modifies Bob's message  $\{Doc, Sig\}$  and  $X_B$  is at most  $(2b_H + b_M)/2^{b'_H-1}$  and  $3b_H/2^{b'_H-1}$ , respectively. Similarly, the probability of a successful attack on the message  $X_C$  from Charlie is  $3b_H/2^{b'_H-1}$ . Since the protocol aborts when Alice fails to tamper with the messages exchanged by Bob and Charlie over the authenticated channel, the maximum probability that the protocol does not abort and Alice successfully changes an authenticated message –thereby causing a repudiation—is given by:  $\varepsilon_{\rm rep} = \max\{(2b_H + b_M)/2^{b'_H-1}, 3b_H/2^{b'_H-1}\}$ , which concludes the proof.

Although forgery attacks in Protocol 1 are related to attacks in WC authentication, there is an important difference. While in WC authentication a given hash function can be reused for subsequent messages (key recycling) when the tags are protected by OTP, in Protocol 1 this is not possible since the hash function becomes known to Bob after the protocol execution and Bob is a potential adversary. This implies that Alice must employ a new function from  $\mathcal{F}_{\text{AXU}}$  for every new signature, i.e., both the initial vector  $X_A^{b_H}$  and the irreducible polynomial  $p_a$  must be renewed.

In the following we prove that renewing  $X_A^{bH}$  without renewing  $p_a$  opens a security loophole. Indeed suppose Alice chooses the same value for  $p_a$  to sign multiple documents. Then, Bob will know  $p_a$  when he receives the second document signed by Alice. This enables Bob to forge a  $\{Doc', Sig'\}$  pair that will be validated by Charlie, thereby making the protocol vulnerable to forgery attacks.

**Lemma III.3.** If  $p_a$  is known to Bob, then he can always perform a successful forgery attack, i.e., Charlie validates the pair  $\{Doc', Sig'\}$  forged by Bob with unit probability.

Proof. To see this, we observe that Bob's goal is to find a  $b_M$ -bit message m and a hash t such that:  $t = T_{p_a, X_A^{b_H}} \cdot m$ . If he succeeds, he can send  $\{Doc', Sig'\}$  with  $Doc' = Doc \oplus m$  and  $Sig' = Sig \oplus (t||0)$ , which will be validated by Charlie, since the expected hash by Charlie is:  $h_c = T_{p_a, X_A^{b_H}} \cdot (Doc \oplus m) = h_a \oplus t$ , which coincides with the hash sent by Bob in Sig'. Now, from the knowledge of  $p_a$ , it is easy for Bob to find a valid hash t and message m

that satisfy  $t=T_{p_a,X_A^{b_H}}\cdot m$ . For instance, Bob can choose t=0 (the null vector). From Theorem 8 in Ref. [20], we have that:

$$\begin{split} t &= T_{p_a, X_A^{b_H}} \cdot m \\ &= BDB^{-1} \cdot X_A^{b_H}, \end{split} \tag{3}$$

where B is a non-singular  $b_H \times b_H$  matrix that only depends on  $p_a$  and D is a diagonal matrix with elements  $m(\lambda_i)$ , where m(x) is the polynomial associated to the message m and  $\lambda_1, \lambda_2, \dots, \lambda_{b_H}$  are the  $b_H$  roots of the irreducible polynomial  $p_a(x)$  over  $GF(2^{b_H})$ . Having fixed t=0, it is enough for Bob to find a polynomial m(x) such that  $BDB^{-1}$  is the null matrix, i.e. such that  $m(\lambda_i) = 0$ for every i. In other words, Bob wants that every root of  $p_a(x)$  (over  $GF(2^{b_H})$ ) is also a root of m(x). This can be readily achieved by choosing a message m such that  $p_a(x)$  divides m(x), i.e. a polynomial that can be decomposed as:  $m(x) = p_a(x) \cdot g(x)$ , where g(x) is an arbitrary polynomial over GF(2) of degree  $b_M - b_H$ . Then, the pair t=0 and the message associated to the polynomial  $m(x) = p_a(x) \cdot g(x)$  are such that  $t = T_{p_a, X_A^{b_H}} \cdot m$  holds, which concludes the proof.

### B. QDS by García Cid et al.

In Ref. [10], García Cid et al. propose a tripartite QDS scheme that combines QKD symmetric keys with NIST-recommended hash functions, which, however, are not IT secure, thus preventing the whole QDS scheme from being so. To make a fair comparison with the other QDS protocols addressed in this manuscript, we describe a modified version of the protocol from [10], where we replace the computationally-secure hash functions with the  $\mathcal{F}_{\rm AXU}$  family of hash functions described in Sec. II. Moreover, we require that Bob forwards the document-signature pair to Charlie via an authenticated channel, otherwise Alice could easily enforce a repudiation.

We point out that, in a recent work [23], the authors modified the original protocol from [10] in order to increase its security and efficiency. Although the scheme from Ref. [23] closely resembles the version we propose below, it is still not IT-secure as successful forgery attacks are always possible by exhaustive search, while this is not possible in our modified version.

# 1. The modified protocol

Similarly to the protocol by Yin et al., Alice establishes symmetric keys with Bob and Charlie with QKD. However, rather than combining the two keys via the XOR operation, she concatenates the two keys into a single key. Similarly to the protocol by Amiri et al., the protocol requires Bob and Charlie to communicate via authenticated secret channels and exchange a random

subset of their symmetric keys. In this way, Alice cannot easily force a repudiation attack by adding noise in the signature destined to one specific receiver, since she does not know anymore the key held by each receiver.

# **Protocol 2** QDS protocol (modified from [10])

- 1. Distribution stage Alice establishes symmetric keys with Bob and Charlie. Bob and Charlie exchange random key blocks.
- 1.1. Alice runs a QKD protocol with Bob (Charlie) and establishes a shared secret key  $X_B$  ( $X_C$ ) of length  $3nb_H$  bits, which can be divided into n blocks of length  $3b_H$  each:  $X_B = X_B^1 || X_B^2 || \dots || X_B^n$  for Bob's key and as  $X_C = X_C^1 || X_C^2 || \dots || X_C^n$  for Charlie's key, where the "||" symbol indicates concatenation. Each block in Bob's key (Charlie's key) can be decomposed as:  $X_B^j = s_B^j || r_B^j (X_C^j = s_C^j || r_C^j)$ , where  $s_B^j (s_C^j)$  is  $b_H$  bits long and  $r_B^j (r_C^j)$  is  $2b_H$  bits long.
- 1.2. Bob (Charlie) selects a random permutation  $\gamma_B \in S_n$  ( $\gamma_C \in S_n$ ) from the set  $S_n$  of permutations of n elements and applies the permutation on their key blocks. Bob obtains the reshuffled key:  $X_B' := X_B^{\gamma_B(1)} ||X_B^{\gamma_B(2)}|| \dots ||X_B^{\gamma_B(n)}||$  and Charlie obtains the reshuffled key:  $X_C' := X_C^{\gamma_C(1)} ||X_C^{\gamma_C(2)}|| \dots ||X_C^{\gamma_C(n)}||$ .
- 1.3. Bob sends the first n/2 blocks of  $X_B'$  to Charlie through the authenticated secret channel, together with their positions:  $\gamma_B(1), \ldots, \gamma_B(n/2)$ . The positions can be encoded into  $(n/2) \log_2 n$  bits sent over the authenticated secret channel. Likewise, Charlie sends the first n/2 blocks of  $X_C'$  to Bob, together with the positions  $\gamma_C(1), \ldots, \gamma_C(n/2)$ .
- 2. Messaging stage Alice sends the signed message to Bob, who verifies it and forwards its to Charlie for verification.
- 2.1. The message Doc is signed by 2n different hash functions from  $\mathcal{F}_{AXU}$ , generating 2n signatures. Each hash function is obtained from a block of Bob's or Charlie's key. For generating the j-th signature  $(1 \leq j \leq n)$ , Alice generates an LFSR-based Toeplitz matrix  $T_{p_B^j,s_B^j}$ , i.e. an element of  $\mathcal{F}_{\text{AXU}}$ (see Appendix B), where the initial vector  $\boldsymbol{s}_{B}^{j}$  is taken from the j-th block of Bob's key  $X_B$  and the irreducible polynomial  $p_B^j$  is obtained by randomly selecting a  $b_H$ -bit string and by checking that the corresponding polynomial over GF(2),  $p_B^j(x) =$  $x^{b_H} + (p_B^j)_{n-1} x^{b_H-1} + \dots + (p_B^j)_1 x + (p_B^j)_0$ , is irreducible (see algorithm in Supplementary Material of [9]). Similarly, for signing the n+j-th block, Alice generates the Toeplitz matrix  $T_{p_C^j, s_C^j}$ , where  $p_C^j(x)$ is an irreducible polynomial of degree  $b_H$  and  $s_C^j$  is taken from the j-th block of Charlie's key  $X_C$ .

2.2. Alice generates the signature  $Sig = Sig_1||...||Sig_{2n}$  by repeatedly hashing the document and encodes the hashes with the  $r_B^j$  and  $r_C^j$  keys from Bob's and Charlie's keys. The first n signatures are given by:

$$Sig_j := (T_{p_B^j, s_B^j} \cdot Doc||p_B^j) \oplus r_B^j \quad j = 1, \dots, n, \quad (4)$$

while the following n signatures are given by:

$$Sig_{j+n} := (T_{p_C^j, s_C^j} \cdot Doc||p_C^j) \oplus r_C^j \quad j = 1, \dots, n.$$
 (5)

Alice sends the pair  $\{Doc, Sig\}$  to Bob over a public channel.

- 2.3. Bob discards all the signatures except those that he can legitimately verify, i.e. the 3n/2 signatures for which he holds the decryption key, namely,  $Sig_1, \ldots, Sig_n$  and  $Sig_{n+\gamma_C(1)}, \ldots, Sig_{n+\gamma_C(n/2)}$ . For each signature  $Sig_j$ , with  $1 \leq j \leq n$ , Bob uses  $X_B^j$  to recover the hash of the document and the irreducible polynomial, by computing  $Sig_j \oplus r_B^j = (h'_j||p'_B^j)$ . Then, he verifies the signature by checking whether  $h'_j = T_{p'_B^j, s_B^j} \cdot Doc$  is satisfied. If not, Bob rejects Alice's signature and sends the symbol  $\bot$  to Charlie over an authenticated channel. Analogously, for each signature of the form  $Sig_{n+\gamma_C(i)}$ , Bob uses the key  $X_C^{\gamma_C(i)}$  received from Charlie to recover the hash and the irreducible polynomial:  $Sig_{n+\gamma_C(i)} \oplus r_C^{\gamma_C(i)} = (h'_{n+\gamma_C(i)}||p'_C^{\gamma_C(i)})$ . Now, Bob verifies that  $h'_{n+\gamma_C(i)} = T_{p'_C^{\gamma_C(i)}, s_C^{\gamma_C(i)}} \cdot Doc$ . If this is not true, Bob aborts and sends  $\bot$  to Charlie. If Bob finds no error during verification, he accepts Doc as original and forwards the pair  $\{Doc, Sig\}$  to Charlie over an authenticated channel.
- 2.4. Charlie verifies the signatures independently of Bob in an analogous way. In particular, he only verifies the signatures  $Sig_{n+1}, \ldots, Sig_{2n}$  and  $Sig_{\gamma_B(1)}, \ldots, Sig_{\gamma_B(n/2)}$  for which he holds the decryption keys. If Charlie observes more than  $e_{max}$  mismatches out of the 3n/2 verified signatures, he rejects Alice's signature, otherwise he accepts the document as original.

## 2. Security proof

We prove the security of the modified protocol that we introduced, Protocol 2, against forgery and repudiation. In doing so, we account for the potential failure of the authenticated channel between Bob and Charlie.

**Lemma III.4.** Protocol 2 is  $\varepsilon_{\text{for}}$ -secure against forgery according to Definition II.1, with

$$\varepsilon_{\text{for}} = \Xi(n/2 - e_{max}, n/2, b_M 2^{1-b_H}),$$
 (6)

where

$$\Xi(k, n, p) := \sum_{j=k}^{n} \binom{n}{j} p^{j} (1-p)^{n-j}.$$
 (7)

*Proof.* There are two cases of forgery attacks: 1) Alice does not sign any document and Bob forges a new pair  $\{Doc', Sig'\}$ ; 2) Alice sends  $\{Doc, Sig\}$  to Bob and Bob forges a modified pair  $\{Doc', Sig'\}$ .

Let us first discuss case 1). In this case, Bob generates a document Doc' and needs to provide Charlie with the correct signature. In particular, since Charlie only verifies a subset of signatures, given by:  $Sig'_{n+1}, \ldots, Sig'_{2n}$  and  $Sig'_{\gamma_B(1)}, \ldots, Sig'_{\gamma_B(n/2)}$ , Bob needs to provide valid signatures for this subset. For the signatures  $Sig'_{\gamma_B(1)}, \ldots, Sig'_{\gamma_B(n/2)}$ , Bob knows the corresponding decryption keys from  $X_B$ , in particular the strings  $r_B^{\gamma_B(i)}$ , which he passed to Charlie in the distribution stage. Thus, Charlie will see no error when verifying these signatures.

In contrast, for the n signatures  $Sig'_{n+1},\ldots,Sig'_{2n}$ , Bob only knows n/2 decryption keys, i.e. the keys  $X_C^{\gamma_C(i)}$  (for  $i=1,\ldots,n/2$ ) that he received from Charlie. In particular, Bob does not know the strings  $r_C^j$  for the n/2 keys that he did not receive from Charlie and needs to guess them in order for Charlie to approve his forged signatures. However, we recall that Charlie is allowed up to  $e_{max}$  errors during verification. Therefore, Bob only needs to correctly guess the string  $r_C^j$  at least  $n/2-e_{max}$  times out of n/2 possibilities. Since each string  $r_C^j$  is  $2b_H$  bits long, the probability for  $k \geq n/2-e_{max}$  successes out of n/2 trials reads:

$$\Xi(n/2 - e_{max}, n/2, 2^{-2b_H}),$$
 (8)

where the function  $\Xi(k,n,p)$  in (7) represents the probability of obtaining at least k successes in n trials with success probability p for each trial. Thus, the quantity in (8) represents the probability of a successful forgery attack by Bob for case 1).

We now discuss case 2). Here, Bob receives a valid pair  $\{Doc, Sig\}$  from Alice and wants to forge a new pair  $\{Doc', Sig'\}$  where  $Doc' \neq Doc$ . Like before, Bob can produce new valid signatures  $Sig'_{\gamma_B(1)}, \ldots, Sig'_{\gamma_B(n/2)}$  since Bob holds the corresponding keys and overrides Alice's choices of irreducible polynomials with his choices of polynomials. Likewise, Bob can forge the signatures  $Sig'_{n+\gamma_C(i)}$  corresponding to the keys  $X_C^{\gamma_C(i)}$  (for  $i=1,\ldots,n/2$ ) that he received from Charlie.

For the remaining n/2 signatures  $Sig_{j+n}$  received from Alice for which Bob does not hold the corresponding key, Bob has two possibilities.

He can act as in case 1), by neglecting the signatures sent by Alice and forging new signatures. However, this strategy requires Bob to guess some of the encryption keys  $r_C^j$ . The probability of correctly guessing one of these keys is  $2^{-2b_H}$ .

The second possibility is also Bob's best strategy since it succeeds with a higher probability: Bob wants to guess a message-tag pair  $(m_j,t_j)$ , with  $m_j \neq 0$ , such that  $t_j = T_{p_C^j,s_C^j} \cdot m_j$  is satisfied. In doing so, note that Bob does not know either parameter of the Toeplitz matrix  $T_{p_C^j,s_C^j}$ 

and that the tag in  $Sig_{j+n}$  is protected by OTP. If Bob succeeds, he can forge the signature  $Sig'_{j+n} = Sig_{j+n} \oplus (t_j||0)$  and choose the document  $Doc' = Doc \oplus m_j$  such that Charlie will validate the forged signature with unit probability. The probability that Bob finds the correct pair  $(m_j, t_j)$  is the probability of a successful attack on WG authentication implemented with the  $\mathcal{F}_{\text{AXU}}$  family and reads  $b_M/2^{b_H-1}$  (see Table I), where  $b_M$  is the length of the document. Let us assume that Bob chooses the same message  $m_1 = m_2 = \cdots = m$  for each of the n/2 signatures, such that the forged document is uniquely defined by  $Doc' = Doc \oplus m$ . Then, the probability that Bob correctly guesses at least  $n/2 - e_{max}$  pairs  $(m, t_j)$ , out of n/2 signatures, is given by:

$$\Xi(n/2 - e_{max}, n/2, b_M 2^{1-b_H}),$$
 (9)

and represents the probability of a successful forgery attack in the case 2).

By comparing the probability of successful attacks from cases 1) (8) and 2) (9), we deduce that the attack from case 2) is always more likely to occur since  $b_M 2^{1-b_H} > 2^{-2b_H}$ . This concludes the proof.

**Lemma III.5.** Protocol 2, with  $b_M + 4nb_H > (n/2)\log_2 n$ , is  $\varepsilon_{\text{rep}}$ -secure against repudiation according to Definition II.2, with

$$\varepsilon_{\text{rep}} = \max \left\{ \prod_{i=0}^{e_{max}} \frac{n/2 - i}{n - i}, \frac{b_M + 4nb_H}{2^{b'_H - 1}} \right\}.$$
(10)

Proof. Let us first assume that the authenticated channel between Bob and Charlie is ideal. In this case, Alice's only chance to force a rejection at Charlie is to deliberately introduce errors in some of the signatures verified by Charlie, namely  $Sig_{n+1}, \ldots Sig_{2n}$ , such that Charlie observes  $e_{max}+1$  errors in verification, thus rejecting the signature. However, Bob verifies n/2 of these signatures, namely  $Sig_{n+\gamma_C(1)}, \ldots, Sig_{n+\gamma_C(n/2)}$ , through the keys  $X_C^{\gamma_C(i)}$  (for  $i=1,\ldots,n/2$ ) received from Charlie. Since Bob accepts no error in verification and since the n/2 key blocks sent by Charlie to Bob are randomly selected, the probability that Alice distributes the  $e_{max}+1$  errors in the n/2 signatures that are not verified by Bob is given by:

$$\varepsilon = \frac{\binom{n/2}{e_{max}+1}}{\binom{n}{e_{max}+1}}$$

$$= \prod_{i=0}^{e_{max}} \frac{n/2-i}{n-i}.$$
(11)

Note that if only one error is introduced by Alice in the signatures verified by Bob, Bob will reject the signature and the protocol aborts.

Let us now consider the failure of the authenticated channel between Bob and Charlie, which we recall is implemented with an  $\varepsilon$ -AXU<sub>2</sub> family based on LFSRs with

hashes of  $b'_H$  bits. In order to force a repudiation, Alice can attempt to modify enough key blocks sent to Charlie by Bob. The probability that Alice successfully modifies the key blocks from  $X'_B$  sent by Bob through the authenticated channel is at most  $(3b_H n/2)/2^{b'_H-1}$ . Alternatively, Alice can modify the positions  $\gamma_B(1), \ldots, \gamma_B(n/2)$  sent by Bob, such that Charlie will hold the right blocks but in the wrong positions. In this case, the probability of a successful attack is  $((n/2)\log_2 n)/2^{b'_H-1}$ . Finally, Alice could attempt to modify the pair  $\{Doc, Sig\}$  when forwarded by Bob, with a success probability of at most  $(b_M + 4nb_H)/2^{b'_H-1}$ . Since the protocol aborts when a message from Bob is not authenticated by Charlie, the maximum probability of a successful repudiation attack based on the failure of authentication is:

$$\varepsilon' = \frac{\max \left\{ 3b_H n/2, (n/2) \log_2 n, b_M + 4nb_H \right\}}{2^{b'_H - 1}}$$
$$= (b_M + 4nb_H)/2^{b'_H - 1}. \tag{12}$$

By combining (11) and (12), the maximal probability that the protocol does not abort and that Charlie rejects the signature while Bob does not is  $\max\{\varepsilon, \varepsilon'\}$ , which concludes the proof.

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The authors in [11] construct an IT-secure QDS scheme with N+1 participants, i.e. a sender and N receivers,  $P_1, \ldots, P_N$ . The protocol is guaranteed to be secure if there is an honest majority of participants and it does not require a fixed party to be honest.

In the N-partite scenario, the concept of security against repudiation is more nuanced. Informally, the QDS protocol guarantees that if an honest receiver accepts a signature, then any other other honest receiver accepts it with high probability; this property is called transferability. However, there are scenarios where transferability cannot be guaranteed. In these cases, the validity of a signature is collectively established by a majority vote resolution process. A successful repudiation attack would cause the majority vote process to reject a signature that was previously accepted by an honest receiver.

Nevertheless, when analyzing the same tripartite scenario of the other two QDS protocols, i.e. N=2, the concepts of transferability and non-repudiation basically reduce to the same requirement, namely, the sender cannot force the two receivers to disagree on the validity of the signature.

# 1. The protocol

The main idea of the protocol is the following. In the distribution stage, the sender distributes to each receiver a subset of hash functions from the  $\mathcal{F}_{ASU}$  family of Sec. II

(see Appendix B for more details), through pairwise secret channels. Since this communication is secret, no receiver can play the part of the sender (security against forgery). Afterwards, each receiver partitions the set of received hash functions and sends different partitions to each other receiver through authenticated secret channels. This step shuffles around the set of hash functions held by each receiver, thus preventing the sender from knowing the hash functions held by a given receiver.

In the messaging stage, the sender signs a document with all the hash functions previously distributed to the receivers and appends the corresponding tags. Each receiver counts the number of mismatches between the hash values obtained with their set of hash functions and the corresponding tags, and only accepts the document if the number of mismatches is below threshold. Since the sender does not know the set of hash functions held by a given receiver, they cannot force certain receivers to reject and others to accept the signature (security against repudiation/transferability).

However, in a N-user scenario, the sender could collude with a subset of malicious receivers. In that case, the colluding coalition could determine a subset of the hash functions held by a given receiver, thus causing mismatches between the tags and their computed hash values. In order to still guarantee transferability, the authors introduce verification levels, which correspond to different error thresholds: the lower the verification level, the higher the error threshold. The protocol guarantees that if a document is verified by an honest receiver at verification level l, any other honest receiver will verify the signature at level l-1, with high probability.

We now formally describe the protocol. The protocol fixes a maximum number of dishonest participants it can tolerate,  $\omega$ , which must be inferior to the majority of participants:  $\omega < (N+1)/2$ . The maximum fraction of dishonest receivers it can tolerate when colluding with the sender is  $d_R$  and is given by:  $d_R = (\omega - 1)/N$ . Finally, the protocol fixes the maximum verification level at  $l_{max}$ , where  $(l_{max} + 1)d_R < 1/2$ .

Importantly, we explicitly specify which protocol steps require authenticated communication, a fact which is not clear from the original formulation of the protocol [11]. This helps clarifying the actual amount of resources (e.g. preshared key bits) consumed by the protocol, but also closes potential security loopholes preventing transferability and non-repudiation.

# Protocol 3 QDS protocol [11]

- 1. Distribution stage The hash functions randomly selected by the sender are distributed to the receivers.
- 1.1. The sender selects uniformly at random (and with replacement)  $N^2k$  hash functions  $(f_1, \ldots, f_{N^2k})$  from  $\mathcal{F}_{ASU}$  (see Appendix B), where k is a security parameter.
- 1.2. Each receiver  $P_i$  receives via a secret channel the Nk functions:  $(f_{(i-1)Nk+1}, \ldots, f_{iNk})$ .

- 1.3. Each receiver  $P_i$  randomly partitions the set of indexes corresponding to the received hash functions,  $\{(i-1)Nk+1,\ldots,iNk\}$ , into N sets of size k denoted  $R_{i\to 1},\ldots,R_{i\to N}$ . The receiver then sends the set  $R_{i\to j}$  and the set of corresponding hash functions,  $F_{i\to j}:=\{f_r:r\in R_{i\to j}\}$ , to  $P_j$ , using authenticated secret channels [Note that the sets  $R_{i\to i}$  and  $F_{i\to i}$  are kept by  $P_i$ ]. In this way, each participant  $P_i$  holds Nk functions, given by:  $F_i:=\bigcup_{j=1}^N F_{j\to i}$ , and their positions, given by:  $R_i:=\bigcup_{j=1}^N F_{j\to i}$ .
- 2. Messaging stage The sender sends the signed message and each receiver independently verifies the signature.
- 2.1. The sender sends the pair  $\{Doc, Sig\}$  to the desired recipient,  $P_i$ , over a public channel, where

$$Sig := (f_1(Doc), f_2(Doc), \dots, f_{N^2k}(Doc))$$
  
=  $(t_1, \dots, t_{N^2k}).$  (13)

2.2. Participant  $P_i$  verifies the signature Sig received by the sender for decreasing verification levels l, until it either accepts the signature or it runs out of levels (l = 0) is the last level of verification).

Initially,  $P_i$  sets the verification level to  $l = l_{max}$ . For a fixed j, participant  $P_i$  tests if the k tags generated from the hash functions received by  $P_j$  match the tags received by the sender. Thus, receiver  $P_i$  defines the test:

$$T_{i,j,l}^{Doc} = \begin{cases} 1 & \text{if } \sum_{r \in R_{j \to i}} g(t_r, f_r(Doc)) < s_l k \\ 0 & \text{else} \end{cases}$$
 (14)

where g(x,y) = 0 (g(x,y) = 1) if x = y  $(x \neq y)$  and  $1/2 > s_{-1} > s_0 > \cdots > s_{l_{max}} > 0$  are parameters fixed by the protocol. The test is passed if  $T_{i,i,l}^{Doc} = 1$ .

The participant  $P_i$  performs the tests  $T_{i,j,l}^{Doc}$  for each j at verification level l. The output of the verification at level l is given by:

$$\operatorname{Ver}_{i,l}(Doc, Sig) = \begin{cases} \operatorname{True} & \text{if } \sum_{j=1}^{N} T_{i,j,l}^{Doc} > N\delta_{l} \\ \operatorname{False} & \text{else} \end{cases}$$
 (15)

where  $\delta_l = 1/2 + (l+1)d_R$ . If the verification failed  $(\text{Ver}_{i,l}(Doc, Sig) = \text{False})$ , the participant  $P_i$  decreases by one the verification level and attempts a new verification. The verification process ends when  $\text{Ver}_{i,l}(Doc, Sig) = \text{True for some } l \geq 0$ , in which case participant  $P_i$  accepted the signature at level l. Otherwise,  $P_i$  rejected the signature.

2.3. If the receiver  $P_i$  accepted the signature at some level  $l \geq 0$ , they forward the pair  $\{Doc, Sig\}$  to the next participant via an authenticated channel. Otherwise, the receiver forwards  $\perp$  to all remaining participants, flagging that the signature was rejected.

### 2. Security definitions

Here we report and comment on the security definitions adopted in Ref. [11]. They can be seen as a generalization of Definitions II.1 and II.2 presented in Sec. II.

**Definition III.1.** [Forgery] Let  $C \subset \{P_1, ..., P_N\}$  be a coalition of malicious users, not including the sender, which knows a valid pair  $\{Doc, Sig\}$ . Then, the QDS protocol is  $\varepsilon_{for}$ -secure against forgery if, for every forged pair  $\{Doc', Sig'\}$  produced by C with  $Doc' \neq Doc$ , it holds:

$$\Pr\left[\exists P_i \notin C : \operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] \leq \varepsilon_{\text{for}}.$$
 (16)

**Definition III.2.** [Non-transferability] Let  $C \subset \{P_0, P_1, \ldots, P_N\}$  be a coalition of malicious users including the sender. Then, the QDS protocol is  $\varepsilon_{\text{transf}}$ -secure against non-transferability if, for every pair  $\{Doc, Sig\}$  produced by C and every verification level  $l \geq 1$ , it holds:

$$\Pr\left[\exists P_i, P_j \notin C : \operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{True} \right. \\ \left. \wedge \operatorname{Ver}_{i,l'}(Doc, Sig) = \operatorname{False} \right] \le \varepsilon_{\operatorname{transf}}, \tag{17}$$

where  $0 \le l' < l$ .

We observe that, according to the description of Protocol 3, if a signature is verified at level l, then it would be verified also at any lower level. That is,

$$\operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{True} \implies$$

$$\operatorname{Ver}_{i,l'}(Doc, Sig) = \operatorname{True}, \ \forall \ l' < l.$$
(18)

Thus, the condition in (17) can be checked for l' = l - 1 only.

Moreover, we argue that the transferability definition reported in Definition III.2 does not capture transferability in a meaningful way. Indeed, it only requires that if an honest receiver accepts a message-signature pair  $\{Doc, Sig\}$ , then another honest receiver accepts the same pair  $\{Doc, Sig\}$  with high probability. The definition does not account for attacks that may modify the pair  $\{Doc, Sig\}$  to be verified by the second receiver. This attack is easily carried out by, e.g., a dishonest receiver who forwards a modified pair to an honest receiver. The protocol has no way to avoid this attack from happening. Alternatively, an attacker could alter the  $\{Doc, Sig\}$  pair in the transmission between two honest receivers, by attacking their authenticated channel. We remark that, in the original paper [11], this channel is not explicitly required to be authenticated, hence the attack would always succeed.

When there are disputes caused e.g. by the sender refusing to recognize a signature validated by a legitimate receiver, the participants can invoke a majority vote dispute resolution method, MV(Doc,Sig), whose outcome is the official accepted outcome. A repudiation attack is successful if the outcome of the majority vote dispute resolution contradicts the outcome of an honest receiver that accepted the signature.

**Definition III.3.** [Repudiation] Let  $C \subset \{P_0, P_1, \ldots, P_N\}$  be a coalition of malicious users including the sender. Then, the QDS protocol is  $\varepsilon_{\text{rep}}$ -secure against repudiation if, for every pair  $\{Doc, Sig\}$  produced by C and every verification level  $l \geq 0$ , it holds:

$$\Pr\left[\exists P_i \notin C : \operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{True} \right. \\ \wedge \operatorname{MV}(Doc, Sig) = \operatorname{Invalid}\right] \leq \varepsilon_{\operatorname{rep}}.$$
 (19)

The  $\mathrm{MV}(Doc,Sig)$  method is invoked, for example, when a receiver validates a signature at verification level l=0 (this can happen if a malicious coalition forces the failure of all other verification levels, except at l=0) and wishes that other receivers would validate the signature as well. The protocol, however, does not guarantee that the next receiver will validate the signature, since transferability is only guaranteed up to level l=1 (c.f. Definition III.2). In this case, the  $\mathrm{MV}(Doc,Sig)$  allows the receiver to gain confirmation from the other users that the signature is authentic.

The majority vote dispute resolution method is defined following the the original protocol [11],

in contrast to the amendment made in Ref. [15], where the verification level used in the dispute resolution is l = 0, that is,  $Ver_{i,-1}$  is replaced by  $Ver_{i,0}$ .

In Ref. [15], the authors argue that a security loophole arises if the verification is carried out at level l=-1, since this case is not guaranteed secure against forgery (c.f. Definition III.1). A malicious receiver could forge a signature and claim that they verified it at level l=0, thus invoking the dispute resolution method, hoping that the signature is accepted by the majority of users if the verification occurs at level l=-1. Indeed, Definition III.1 does not guarantee that the protocol can detect forgeries when the forged signature is verified at l=-1.

As said, the authors in Ref. [15] close the loophole by raising the verification level of the dispute resolution method from l=-1 to l=0. However, in doing so, they lose the original meaning of security against repudiation. Indeed, now it could happen that the first receiver of a pair  $\{Doc, Sig\}$  verifies at level l=0 and whishes to invoke MV(Doc, Sig). Since the verification therein occurs at the same level as the first receiver, the protocol cannot guarantee that the dispute resolution will agree with the first user with high probability – as a matter of fact, transferability is only guaranteed between different verification levels (c.f. Definition III.2). In other words, security against repudiation, as defined in Definition III.3, is lost.

To avoid losing this aspect of the protocol, we solve the loophole issue by computing the probability of success of the above-described forgery attack and by showing that it remains small and of the order of  $\varepsilon_{\rm for}$  in the scenarios of interest, thus removing the need for a redefinition of (20) (see Lemma III.6).

## 3. Security proof

We prove the security of Protocol 3 with respect to Definitions III.1, III.2, and III.3, improving the security parameters where possible while accounting for the failure of IT-secure authenticated channels, unlike the original paper [11]. In particular, a transferability attack is enabled by altering the hash functions destined to a given participant via successful attacks on the authenticated channels in step 1.3. Note that, in the original paper [11], these channels are not explicitly required to be authenticated.

In the proofs, we assume that the protocol parameters  $s_{-1}, s_0, \ldots, s_{l_{max}}$  are equally spaced in the interval [0, 1/2], with spacing  $\Delta s = s_{l-1} - s_l$ , and we assume that  $\Delta s$  is maximal. This choice is optimal to reduce the attack probability on transferability and non-repudiation [11]. Moreover, we note that the constraints on the parameters  $s_l$  imply the following constraint on the spacing:  $(l_{max} + 1)\Delta s < 1/2$ .

**Lemma III.6.** Protocol 3 is  $\varepsilon_{\text{for}}$ -secure against forgery according to Definition III.1, with:

$$\varepsilon_{\text{for}} = (N - \omega) \Xi(|N/2|, N - \omega, p_t),$$
 (21)

where  $p_t$  is defined as:

$$p_t = \Xi(\lfloor k(1-s_0) \rfloor + 1, k, 2^{1-b_H}),$$
 (22)

Moreover, the forgery attack based on the security loophole discussed in Ref. [15], where a dishonest receiver forges the pair {Doc', Sig'} and invokes the dispute resolution method for the other parties to accept the pair, succeeds with probability:

$$p_{\text{attack}} = \Xi(|N/2| + 1 - \omega, N - \omega, p_{-1}),$$
 (23)

with:

$$p_{-1} = \Xi(|N/2| + 1 - \omega, N - \omega, p_t). \tag{24}$$

**Lemma III.7.** Protocol 3 is  $\varepsilon_{transf}$ -secure against non-transferability according to Definition III.2 and it is  $\varepsilon_{rep}$ -secure against repudiation according to Definition III.3, with:

$$\varepsilon_{\text{transf}} = \varepsilon_{\text{rep}} = {N(1 - d_R) \choose 2} \max{\{\varepsilon_{i,j}, \varepsilon_{\text{auth}}, \varepsilon_{\text{hyb}}\}},$$
(25)

where

$$\varepsilon_{i,j} = N(1 - d_R) \exp\left(-\frac{k}{8(l_{max} + 1)^2}\right)$$
 (26)

$$\varepsilon_{\text{auth}} = \left(\frac{ky + k \log_2(Nk)}{2^{b'_H - 1}}\right)^{N[1/2 - (l_{max} + 1)d_R]} \tag{27}$$

$$\varepsilon_{\text{hyb}} = \frac{ky + k \log_2(Nk)}{2^{b_H'-1}}$$

$$\Xi\left(\left\lfloor \frac{N}{2} \right\rfloor + 1, N(1 - d_R), \exp\left(-\frac{k}{8(l_{max} + 1)^2}\right)\right). \tag{28}$$

The function  $\Xi(k, n, p)$ , already defined in (7), represents the probability of at least k successes out of n trials with probability p. We present the security proofs of Lemmas III.6 and III.7 in Appendix C.

#### IV. PERFORMANCE OPTIMIZATION

In this section we compare the performance of the three QDS protocols, presented in Sec. III, in a tripartite scenario with one sender and two receivers.

Specifically, given a document of fixed length  $(b_M)$ , we optimize the performance of each protocol over their free parameters, such that the number of preshared secret key bits per receiver  $(\ell_P)$  and the length of the signature  $(\ell_S)$  are minimized, under the constraint that  $\varepsilon_{\text{rep}} + \varepsilon_{\text{for}} \leq 10^{-10}$  (c.f. Definitions II.1 and II.2). In particular, Protocol 1 (based on the protocol by Yin et al. [9]) is optimized over the parameters  $b_H, b'_H$ . Protocol 2 (a modified version of the protocol by García Cid et al. [10]) is optimized over  $n, b_H, b'_H, e_{max}$ . Protocol 3 (based on the protocol by Amiri et al. [11]) is optimized over  $k, b_H, b'_H$ .

In Table II we report the security parameters, the number of preshared key bits per receiver and the signature length of each protocol that are used in our optimizations. Note that Protocol 3 is studied for N=2. In this case, the maximum number of dishonest parties is  $\omega=1$ , the fraction of dishonest receivers that collude with the sender is  $d_R=0$  and the maximum number of transfers between receivers is  $l_{max}=1$ . Moreover, with the optimal choice of spacing between the  $s_l$  variables, we have:  $s_0\approx\frac{1}{2}-\Delta s=\frac{1}{2}-\frac{1}{2(l_{max}+1)}=\frac{1}{4}$ .

It is interesting to notice that, for N=2, the success probability  $(p_{\rm attack})$  of the forgery attack based on the loophole noted in Ref. [15] coincides with the regular forgery probability  $(\varepsilon_{\rm for})$ . Thus, in reality, the attack mentioned in Ref. [15] does not give rise to a security loophole in the analyzed scenario.

### A. Calculation of $\ell_P$

In order to achieve a fair comparison, we consider that every secret channel is implemented via OTP, thus requiring a number of preshared key bits equal to the length of the message. Moreover, every authenticated channel is implemented as described in Sec. II, via WC authentication with key recycling and tags of  $b'_H$  bits. In the following, we illustrate how to obtain the number of preshared key bits  $(\ell_P)$  reported for each protocol in Table II.

Protocol	$igg \ell_P$	$ig _{\ell_S}$	$arepsilon_{ ext{for}}$	$arepsilon_{ m rep}$
Protocol 1 from Ref. [9]	$3b_H + 5b'_H$	$2b_H$	$b_M/2^{b_H-1}$	$(2b_H + b_M)/2^{b_H'-1}$
Protocol 2 modified from Ref. [10]	$6nb_H + n\log_2 n + 7b_H'$	$4nb_H$	$\Xi(n/2 - e_{max}, n/2, b_M 2^{1-b_H})$	$\max \left\{ \prod_{i=0}^{e_{max}} \frac{n/2-i}{n-i}, \frac{b_M + 4nb_H}{2^{b_H'-1}} \right\}$
Protocol 3 from Ref. [11]	$3ky + k\log_2(2k) + 7b_H'$	$4kb_H$	$\Xi\left(\left\lfloor \frac{3}{4}k\right\rfloor + 1, k, 2^{1-b_H}\right)$	$\max \left\{ 2e^{-k/32}, \frac{ky + k \log_2(2k)}{2^{b'_H - 1}} \right\}$

Table II. For each analyzed protocol, we report the security parameters against forgery ( $\varepsilon_{\text{for}}$ ) and repudiation attacks ( $\varepsilon_{\text{rep}}$ ), as well as the consumed preshared secret key bits per receiver ( $\ell_P$ ) and the length of the signature ( $\ell_S$ ). Each protocol is used to sign a document of  $b_M$  bits with hash functions drawn from either the  $\mathcal{F}_{\text{AXU}}$  family (Protocols 1 and 2) or the  $\mathcal{F}_{\text{ASU}}$  family (Protocol 3), producing hashes of  $b_H$  bits. The parameter y represents the number of bits required to specify an element of  $\mathcal{F}_{\text{ASU}}$  and is given in Appendix B. The function  $\Xi$  is given in (7).

**Protocol 1** The protocol requires each receiver to hold a preshared secret key of length  $3b_H$ . Moreover, to establish the authenticated channel between Bob and Charlie, each receiver consumes  $2b'_H$  preshared bits. The three messages that are sent over the authenticated channel amount to additional  $3b'_H$  consumed preshared bits (needed to encrypt the tags).

**Protocol 2** The protocol requires each receiver to share a  $3nb_H$ -bit secret key with Alice. Then, it consumes  $3nb_H + n\log_2 n$  preshared bits to exchange half of the key blocks between Bob and Charlie, together with their positions. To establish the authenticated channel between Bob and Charlie, each receiver consumes  $2b'_H$  preshared bits. Bob and Charlie exchange 4 authenticated messages in the distribution stage; moreover, Bob forwards the  $\{Doc, Sig\}$  pair on the authenticated channel in the messaging stage, for a total of 5 authenticated messages.

**Protocol 3** To carry out step 1.2 of Protocol 3, each receiver must hold Nky preshared key bits with the sender, where y is the number of bits required to specify one element of the  $\mathcal{F}_{ASU}$  family and is provided in Appendix B. For step 1.3 of the protocol, each receiver  $P_i$  sends to another receiver,  $P_j$ , k hash functions from  $\mathcal{F}_{ASU}$  and their positions, thus consuming a total of  $(N-1)(ky+k\log_2(Nk))$  preshared secret bits. Moreover, this communication is also authenticated. Hence, it requires each receiver to share, with any other receiver,  $2b'_H$  bits to agree on the hash function and additional  $4b'_{H}$  bits to exchange the hash functions and their positions. Finally,  $b'_H$  preshared bits are consumed to forward the  $\{Doc, Sig\}$  pair to the next recipient in an authenticated manner. The resulting number of preshared key  $1)6b'_H + b'_H$ . In Table II we specify it for N=2.

## B. Optimization results

The results of our numerical optimizations for the three protocols are presented in Figs. 1 and 2, for documents of  $b_M$  bits, with:  $b_M \in [10^2, 10^{10}]$ .

In Fig. 1 we plot the signature length  $(\ell_S)$  as a function of the document size  $(b_M)$ . We observe that Protocol 1 requires much shorter signatures compared to the other two protocols. Moreover, while Protocol 1 and Protocol 2 require signatures with length scaling linearly in  $\log_2 b_M$  (though with very different slopes), Protocol 3 requires large, but constant, signature lengths.

In Fig. 2 we plot the number of preshared key bits  $(\ell_P)$  as a function of the document size  $(b_M)$ . We observe that all three protocols require a number of preshared bits that scales sub-linearly with respect to  $b_M$ , hence becoming more efficient as the size of the document grows. However, there are important gaps in absolute magnitude between the protocols and Protocol 3 appears to be the least efficient. For instance, to sign a document of  $b_M = 10^6$  bits, Protocol 3 requires  $\ell_P \approx 1.1 \cdot 10^5$  preshared secret bits, while Protocol 2 uses  $\ell_P \approx 10^4$  bits and Protocol 1 only uses  $\ell_P = 441$  bits.

In order to explain the observations regarding Figs. 1 and 2, we analyzed the optimal parameters returned by our optimizations. In particular, in the numerical optimizations for Protocol 1, we obtain the following approximate optimal values for the optimization parameters  $b_H$  and  $b'_H$ :

$$b_H \approx \log_2\left(\frac{b_M}{3.75 \cdot 10^{-11}}\right) + 1$$
 (29)

$$b'_{H} \approx \log_2 \left( \frac{b_M + 2b_H}{6.25 \cdot 10^{-11}} \right) + 1.$$
 (30)

By comparing these values with Table II, we deduce that both  $\ell_P$  and  $\ell_S$  scale logarithmically with the document size,  $\ell_P, \ell_S \sim \log_2 b_M$ , with relatively small prefactors. This explains the superior performance of Protocol 1

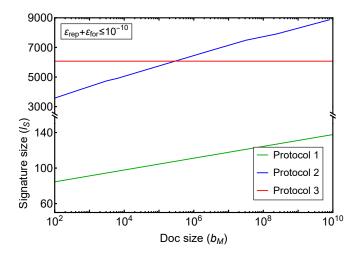


FIG. 1. The optimal signature length  $(\ell_S)$  as a function of the document size  $(b_M)$  for the three analyzed QDS protocols, when minimized under the constraint:  $\varepsilon_{\text{rep}} + \varepsilon_{\text{for}} \leq 10^{-10}$ .

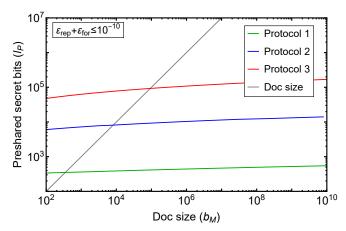


FIG. 2. The optimal number of preshared secret bits  $(\ell_P)$  as a function of the document size  $(b_M)$  for the three analyzed QDS protocols, when minimized under the constraint  $\varepsilon_{\rm rep} + \varepsilon_{\rm for} \leq 10^{-10}$ .

both in terms of consumed preshared keys and signature lengths compared to the other two protocols.

A similar scaling is observed for the optimization parameters  $b_H$  and  $b_H'$  in Protocol 2, i.e.,  $b_H, b_H' \sim \log_2 b_M$ , such that  $\ell_P$  and  $\ell_S$  also scale logarithmically with the document size. However, due to the large values acquired by the parameter  $n \approx 50$ , the resulting  $\ell_P$  and  $\ell_S$  of Protocol 2 display large prefactors ( $\approx 300$  for  $\ell_P$  and  $\approx 200$  for  $\ell_S$ ) compared to Protocol 1. This explains the steep increase in signature length compared to the other two protocols observed in Fig. 1.

In the numerical optimization for Protocol 3, we observe that:

$$b_H = 2 \tag{31}$$

is optimal for every tested document length, recovering the observation made in Ref. [11]. Moreover, we observe that the optimal  $b_H^\prime$  is such that:

$$2e^{-k/32} = \frac{ky + k \log_2(2k)}{2^{b_H'-1}}$$
 (32)

is satisfied, which implies:

$$b'_{H} = \log_2(ke^{k/32}) + \log_2(y + \log_2(2k))$$
. (33)

By making the ansatz that (31) and (33) hold, the optimization of Protocol 3 runs over only one parameter, k, which is fixed by the constraint:

$$\varepsilon_{\text{rep}} + \varepsilon_{\text{for}} = 10^{-10}$$

$$\iff 2e^{-k/32} + \Xi\left(\frac{3}{4}k, k, \frac{1}{2}\right) = 10^{-10}$$

$$\iff k \approx 759.$$
(34)

Recalling that  $y \sim \log_2 b_M$  (see Appendix B), we deduce that  $\ell_P$  scales logarithmically with the document size albeit with a very large prefactor:  $\ell_P \sim 3 \times 759 \log_2 b_M$ , thus explaining the worst performance among the three protocols as noted in Fig. 2. Conversely, the signature length is constant for any document size:  $\ell_S \approx 6072$  as observed in Fig. 1.

In summary, from our performance analysis in the tripartite scenario, we conclude that Protocol 1 [9] is the most efficient protocol in terms of consumed preshared bits, with a consumption up to three orders of magnitude smaller than the other two protocols and in the range  $\ell_P \in [10^2, 10^3]$  for document sizes up to  $b_M = 10^{10}$  bits. At the same time, Protocol 1 is the one that generates signatures with the shortest lengths, scaling with the logarithm of the size of the document and beating the other protocols by more than one order of magnitude.

#### V. CONCLUSION

In this work we investigated three practical quantum digital signature protocols from Refs. [9–11], capable of signing large documents with relatively small signatures, while only requiring previously-established secret keys (e.g. through quantum key distribution). The secret keys are primarily used to agree on hash functions from universal families and to establish secret channels protected by one-time pad.

We carefully reviewed the security of each protocol (and in particular the use of authenticated communication) and made modifications where deemed necessary, in order to prove their information-theoretic security and avoid potential loopholes.

We then numerically optimized each protocol in the tripartite scenario to reduce the consumption of preshared secret bits as well as the signature lengths, for a fixed security threshold. We found that the QDS protocol from Ref. [9] is the most efficient among the three protocols in terms of consumed preshared bits and signature length, both scaling with the logarithm of the size of the document and decreasing by more than one order of magnitude compared to the other protocols.

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### APPENDIX A: Wegman-Carter authentication

Hash functions map larger domains to smaller sets such that, with high probability, if the hashed values of two quantities are equal, then the two quantities are also equal. Authenticated classical channels with information-theoretic (IT) security can be built from  $\varepsilon$ -ASU<sub>2</sub> hash families (Definition II.3) through the Wegman-Carter (WC) authentication method [21] with key recycling. In this Appendix, we first illustrate the WC method with key recycling and then prove its IT security.

# Protocol 4 WC authentication [21]

- 1. Let M be the set of possible messages and B the set of hashes, or tags. Let  $\mathcal{F}$  be a publicly-known  $\varepsilon$ -ASU<sub>2</sub> family of hash functions from M to B.
- 2. Alice and Bob agree on the total number of messages n they want to authenticate and they agree on a uniformly random secret key  $(k_0, (k_1, \ldots, k_n))$ . The subkey  $k_0$  identifies a unique hash function  $f_{k_0} \in \mathcal{F}$  used to generate the tags of each of the n authenticated messages.
- 3. For  $i \in \{1, 2, \dots, n\}$ , do the following:
- (a) Alice chooses a message  $m_i \in M$  to be sent and generates the corresponding tag  $t_i$  as:  $t_i = f_{k_0}(m_i) \oplus k_i$ . She sends the message  $m_i$ , the tag  $t_i$  and the index i to Bob.
- (b) Bob uses the subkey  $k_i$ , corresponding to the received index from Alice, to extract the hash from the received tag by computing:  $h_i = t_i \oplus k_i$ . Bob then compares  $h_i$  with the hash resulting from applying the selected hash function on the received message:  $f_{k_0}(m_i)$ . If they match, Bob authenticates Alice's message  $m_i$ .

Here we prove the IT security of the WC authentication method implemented with an  $\varepsilon$ -ASU<sub>2</sub> family.

**Theorem A.1.** Let  $(k_0, (k_1, ..., k_n))$  be a randomly chosen key known to Alice and Bob and let  $m_1, ..., m_n$  be n messages sent by Alice to Bob via the WC authentication method. Suppose the attacker, Eve, knows the  $\varepsilon$ -ASU<sub>2</sub> family  $\mathcal{F}$ , the set of messages M, the messages  $m_1, ..., m_n$ , their tags  $t_1, ..., t_n$  and indexes. Then, there is no forged message  $m_E$  (for any index i) for which Eve can guess the correct tag  $t_E$  with a probability larger than  $\varepsilon$ .

Proof. Suppose, without loss of generality, that Eve wants to replace the first message  $m_1$  with a forged message  $m_E$ . Then, the correct tag such that Bob authenticates the forged message would be:  $t_E = f_{k_0}(m_E) \oplus k_1$ . Note that Eve does not know the preshared key  $(k_0, (k_1, \ldots, k_n))$ , hence she needs a strategy to correctly guess  $t_E$ . An optimal strategy consists in fixing a value for the tag, t, and consider all the possible keys  $(k_0, (k_1, \ldots, k_n))$  for which the observed message-tag pairs  $(m_i, t_i)$  and the forged pair  $(m_E, t)$  are successfully authenticated. Let us denote this set S(t):

$$S(t) = \{ (k_0, (k_1, \dots, k_n)) : t = f_{k_0}(m_E) \oplus k_1, t_1 = f_{k_0}(m_1) \oplus k_1, t_i = f_{k_0}(m_i) \oplus k_i \text{ for } i = 2, \dots, n \}.$$
(A1)

Then, Eve's guess of the tag corresponds to the value for which there are most key combinations allowed, i.e.,

$$t_{\text{guess}} := \arg\max_{t \in B} |S(t)|, \tag{A2}$$

where |S| is the cardinality of S. Indeed, this guess maximizes Eve's probability of guessing the correct tag. In particular, given messages  $m_1, \ldots, m_n$ , tags  $t_1, \ldots, t_n$  and a forged message  $m_E$ , Eve's guess is deterministic and given by  $t_{\text{guess}}$ . The probability that  $t_{\text{guess}}$  is correct corresponds to the probability that the key  $(k_0, (k_1, \ldots, k_n))$ , randomly chosen by Alice and Bob, is such that  $t_E = f_{k_0}(m_E) \oplus k_1 = t_{\text{guess}}$ , conditioned on the compatibility with the observed message-tag pairs. Then, we have that Eve's guessing probability is given by:

$$p_{\text{guess}} = \Pr_{(k_0, (k_1, \dots, k_n))} [t_E = t_{\text{guess}} | t_i = f_{k_0}(m_i) \oplus k_i \text{ for } i = 1, \dots, n] = \frac{|S(t_{\text{guess}})|}{|\mathcal{F}|},$$
(A3)

where we used the fact that the keys are chosen randomly by Alice and Bob and that their total number (given that  $(m_i, t_i)$  are authenticated) is

$$|\{(k_0, (k_1, \dots, k_n)) : t_i = f_{k_0}(m_i) \oplus k_i \text{ for } i = 1, \dots, n\}| = |\mathcal{F}|.$$
 (A4)

As a matter of fact, once we fix  $k_0$ , then all other keys are fixed by the equations  $k_i = t_i \oplus f_{k_0}(m_i)$ .

In the following, we compute Eve's guessing probability starting from the cardinality of S(t), |S(t)|. For a fixed  $k_1$ , there are at most  $\varepsilon |\mathcal{F}|/|B|$  possible choices of  $f \in \mathcal{F}$  such that  $f(m_E) = t \oplus k_1 \wedge f(m_1) = t_1 \oplus k_1$  due to the property (1) of an  $\varepsilon$ -ASU<sub>2</sub> family. Therefore, there are at most  $\varepsilon |\mathcal{F}|/|B|$  possible values of  $k_0$  for each given  $k_1$ . Now, for each given value of  $k_0$ , there is a unique string of keys  $k_2, \ldots, k_n$  such that the remaining conditions of the set S(t) are satisfied; these keys are fixed by:  $k_i = t_i \oplus f_{k_0}(m_i)$ . Thus, summing up, there are at most  $|B| \cdot \varepsilon |\mathcal{F}|/|B|$  keys in S(t):

$$|S(t)| \le \varepsilon |\mathcal{F}| = |S(t_{\text{guess}})|,$$
 (A5)

where we used that  $t = t_{guess}$  maximizes the size of S(t). By employing (A5) in (A3), we obtain Eve's guessing probability

$$p_{\rm guess} = \varepsilon,$$
 (A6)

which concludes the proof.

### APPENDIX B: Efficient hashing families

In this Appendix we specify the  $\varepsilon$ -ASU<sub>2</sub> family and the  $\varepsilon$ -AXU<sub>2</sub> family adopted in the analyzed QDS protocols. We follow the notation laid out in Sec. II such that each hash function maps strings of  $b_M$  bits to strings of  $b_H$  bits. The chosen families are selected for their efficiency in the number of preshared key bits required to uniquely identify a function of the family. In particular, the number of preshared bits scales with  $\log_2 b_M$  (for a fixed security parameter  $\varepsilon$ ), rather than with  $b_M$  as for strongly-universal<sub>2</sub> sets based on random matrices.

 $\varepsilon$ -ASU<sub>2</sub> family: The  $\varepsilon$ -ASU<sub>2</sub> family employed in this manuscript is  $\mathcal{F}_{ASU}$ , with  $\varepsilon = 2^{1-b_H}$ . The explicit construction of  $\mathcal{F}_{ASU}$  is provided in Refs. [15, 24]. Each function of the set  $\mathcal{F}_{ASU}$  is uniquely defined by a string of y bits, where y satisfies:  $y = 3b_H + 2\sigma$ , where  $\sigma$  is the smallest number that satisfies:  $b_M \leq (b_H + \sigma)(1 + 2^{\sigma})$ . Since  $\sigma$  is defined by a transcendental inequality, it can only be computed numerically. Alternatively, it might be convenient to obtain a relatively tight analytical upper bound on  $\sigma$  by solving:  $b_M = b_H (1 + 2^{\sigma})$ , which yields

$$\bar{\sigma} = \log_2 \left( \frac{b_M}{b_H} - 1 \right). \tag{B1}$$

This provides us with the following upper bound on the number of bits y required to specify an element in  $\mathcal{F}_{ASU}$ ,

$$\bar{y} = \left[ 3b_H + 2\log_2\left(\frac{b_M}{b_H} - 1\right) \right],\tag{B2}$$

and this is the value that we use in our performance analysis of Sec. IV.

 $\varepsilon$ -AXU<sub>2</sub> family: The  $\varepsilon$ -AXU<sub>2</sub> family employed in this manuscript is  $\mathcal{F}_{AXU}$ , with  $\varepsilon = b_M 2^{1-b_H}$ , and it is composed of Toeplitz matrices such that consecutive columns are consecutive states of a linear feedback shift register (LFSR) of length  $b_H$  [20]. Therefore, each hash function, i.e. each Toeplitz matrix, is specified by an LFSR and its initial state, totaling to  $2b_H$  bits.

In order to specify the elements of  $\mathcal{F}_{AXU}$ , we first define LFSRs.

**Definition B.1.** Let p(x) be a polynomial of degree n over GF(2),  $p(x) = x^n + p_{n-1}x^{n-1} + \dots p_1x + p_0$  and  $s^0 = (s_{n-1}, \dots, s_1, s_0)$  a binary string. A linear feedback shift register (LFSR) of length n is specified by p(x) and the initial state of the register,  $s^0$ . The following state of the register,  $s^1$ , is obtained by shifting to the right the bits of the previous state,  $s^0$ , and by computing the new element:  $s_n = s^0 \cdot p \mod 2$ , where p is the vector of coefficients  $p = (p_{n-1}, \dots, p_0)$  and where "·" indicates the scalar product. Thus we have:  $s^1 = (s_n, s_{n-1}, \dots, s_1)$ . By applying the same rule recursively, the k-th state of the LFSR is given by:  $s^k = (s_{n-1+k}, \dots, s_{1+k}, s_k)$ , where a generic element of the sequence of bits generated by the LFSR is:  $s_l = s^{l-n} \cdot p \mod 2$ , for  $l \ge n$ .

From the above definition, we deduce that the transpose of consecutive states of an LFSR can be considered as consecutive columns of a Toeplitz matrix. We now define the elements of  $\mathcal{F}_{AXU}$ .

**Definition B.2.** Let p(x) be an irreducible polynomial of degree  $b_H$  over GF(2) and let  $s^0 = (s_{b_H-1}, \ldots, s_1, s_0)^T$  be the initial state of the LFSR with connection polynomial p(x), with  $s^0 \neq 0$ . Let  $T_{p,s}$  be the Toeplitz matrix with columns given by:  $T_{p,s} = (s^0, s^1, \ldots, s^{b_M-1})$ , where  $s^k$  is the transpose of the k-th state of the LFSR. The hash function  $f_{p,s} \in \mathcal{F}_{\text{AXU}}$  maps messages  $m = (m_0, \ldots, m_{b_M-1})^T$  of variable length up to  $b_M$  bits to  $b_H$ -bit strings given by:  $f_{p,s}(m) = T_{p,s}m \mod 2$ . In other words, the j-th element of the hash reads:  $(f_{p,s}(m))_j = \bigoplus_{i=0}^{b_M-1} m_i \, s_{b_H-j+i}$ , where  $s_{b_H-j+i}$  is the  $(b_H-j+i)$ -th element of the LFSR sequence.

Note that to avoid allowing an adversary to add zeroes to the message without changing the tag, each message needs to end with a 1.

### APPENDIX C: Security proof of Protocol 3

In this Appendix we prove the IT security of the QDS protocol presented in Sec. III C.

**Lemma.** Protocol 3 is  $\varepsilon_{transf}$ -secure against non-transferability according to Definition III.2, with:

$$\varepsilon_{\text{transf}} = \binom{N(1 - d_R)}{2} \max\{\varepsilon_{i,j}, \varepsilon_{\text{auth}}, \varepsilon_{\text{hyb}}\}, \tag{C1}$$

where

$$\varepsilon_{i,j} = N(1 - d_R) \exp\left(-\frac{k}{8(l_{max} + 1)^2}\right) \tag{C2}$$

$$\varepsilon_{\text{auth}} = \left(\frac{ky + k \log_2(Nk)}{2^{b'_H - 1}}\right)^{N[1/2 - (l_{max} + 1)d_R]} \tag{C3}$$

$$\varepsilon_{\text{hyb}} = \frac{ky + k \log_2(Nk)}{2^{b'_H - 1}} \Xi\left(\left\lfloor \frac{N}{2} \right\rfloor + 1, N(1 - d_R), \exp\left(-\frac{k}{8(l_{max} + 1)^2}\right)\right),\tag{C4}$$

and

$$\Xi(k, n, p) := \sum_{j=k}^{n} \binom{n}{j} p^{j} (1-p)^{n-j}.$$
 (C5)

*Proof.* According to the description of Protocol 3, if a signature is verified at level l, then it is also verified also at any lower level:

$$\operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{True} \implies \operatorname{Ver}_{i,l'}(Doc, Sig) = \operatorname{True}, \ \forall \ l' < l.$$
 (C6)

By the contrapositive we have that:

$$\operatorname{Ver}_{i,l'}(Doc, Sig) = \operatorname{False} \implies \operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{False} \quad \forall l > l'.$$
 (C7)

Thus, we can restrict without loss of generality the condition in Definition III.2 to verifying that

$$\Pr\left[\exists P_i, P_i \notin C : \operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{True} \wedge \operatorname{Ver}_{i,l-1}(Doc, Sig) = \operatorname{False}\right] \leq \varepsilon_{\operatorname{transf}},$$
 (C8)

where we consider the largest possible coalition C of dishonest users, formed by the dishonest sender and  $Nd_R$  dishonest receivers.

To start with, we fix the pair of honest receivers that are targeted by the coalition to be  $P_i$  and  $P_j$ , respectively. Nevertheless, the attack is successful whenever any pair of honest receivers disagree on the verification outcome; we cover this case at the end of the proof. Moreover, we start by considering the possible attacks from the coalition C that do not involve attacking the authenticated channels and are instead based on distributing invalid hash functions to honest receivers.

The probability of a successful attack of the coalition on the pair  $P_i, P_j$  is given by:

$$\Pr[\text{attack on } P_i, P_j] = \Pr\left[\text{Ver}_{i,l}(Doc, Sig) = \text{True } \wedge \text{Ver}_{j,l-1}(Doc, Sig) = \text{False}\right]$$

$$= \Pr\left[\sum_{r=1}^{N} T_{i,r,l}^{Doc} > \frac{N}{2} + (l+1)Nd_R \wedge \sum_{r=1}^{N} T_{j,r,l-1}^{Doc} \le \frac{N}{2} + lNd_R\right], \tag{C9}$$

where we used the definitions of the verification function and of  $\delta_l$ . Now, we observe that for the tests  $T_{j,r,l-1}^{Doc}$  where  $r \in C$ , the coalition can force the test to not pass. Indeed, a dishonest receiver r can forward invalid has functions to  $P_j$  such that the number of discrepancies observed by  $P_j$  exceeds the threshold  $s_{l-1}k$ , forcing  $T_{j,r,l-1}^{Doc} = 0$ . At the same time, the coalition can behave honestly with respect to receiver  $P_i$ , making sure that their tests are passed:  $T_{i,r,l}^{Doc} = 1$ . Since there are  $Nd_R$  dishonest receivers, there can be at most  $Nd_R$  tests that are passed by  $P_i$  and not

passed by  $P_j$ . Since this occurs deterministically and cannot be avoided, the condition to be checked in (C9) updates to:

$$\Pr[\text{attack on } P_i, P_j] = \Pr\left[\sum_{h=1}^{N(1-d_R)} T_{i,h,l}^{Doc} > \frac{N}{2} + lNd_R \wedge \sum_{h=1}^{N(1-d_R)} T_{j,h,l-1}^{Doc} \le \frac{N}{2} + lNd_R\right], \tag{C10}$$

where now the sums only run over the set of honest receivers,  $h \notin C$ . At this point, the coalition's ability to steer the result of a test where the hash functions originate from an honest receiver, h, is reduced, but not null. In particular, to each honest receiver h, the dishonest sender can provide a set of Nk hash functions, some of which are correct while others are invalid (in the sense that their hashed values are different from the tags). Then, depending on the random samples of k hash functions drawn by k and sent to participants k and k, the respective tests k and k are k and k and k are k and k are k and k and k and k and k and k are k and k and k and k are k and k and k and k and k are k and k are k and k and k and k are k and k and k are k and k and k are k are k and k are k are k and k are k and

In order to find the optimal strategy for the coalition, we consider a necessary condition for the event in (C10) to occur. Namely, that there exists at least one honest receiver,  $\bar{h}$ , such that  $P_i$ 's test is passed while  $P_j$ 's test fails. Note that this condition may also be sufficient since the event in (C10) occurs as soon as the number of passed tests between  $P_i$  and  $P_j$  differs by one. Since the probability of a necessary condition is larger than the probability of the original event, we have the upper bound:

$$\Pr[\text{attack on } P_i, P_j] \leq \Pr\left[\exists \bar{h} \notin C : T_{i,\bar{h},l}^{Doc} = 1 \land T_{j,\bar{h},l-1}^{Doc} = 0\right] \\
\leq N(1 - d_R) \Pr\left[T_{i,h,l}^{Doc} = 1 \land T_{j,h,l-1}^{Doc} = 0\right] \\
\leq N(1 - d_R) \min\left\{\Pr\left[T_{i,h,l}^{Doc} = 1\right], \Pr\left[T_{j,h,l-1}^{Doc} = 0\right]\right\},$$
(C11)

where in the second inequality we used the union bound and assumed without loss of generality that the probability of mismatching outcomes for participants  $P_i$  and  $P_j$  is independent of the choice of honest receiver h.

As mentioned earlier, the coalition can provide participant h with a set of hash functions where some of the functions are invalid, meaning that their hashed values would differ from the tags sent by the sender. Regardless of the strategy, since participant h randomly samples the sets  $F_{h\to i}$  and  $F_{h\to j}$  to be sent to  $P_i$  and  $P_j$ , respectively, the expected number of mismatches between the tags and the hashed values observed by  $P_i$  and by  $P_j$  coincides. We define the expected fraction of mismatches observed by  $P_i$  and  $P_j$  to be  $p_e$ :

$$p_e = E\left[\sum_{r \in R_{h \to i}} \frac{g(t_r, f_r(Doc))}{k}\right]$$
 (C12)

$$= E\left[\sum_{r \in R_{h \to j}} \frac{g(t_r, f_r(Doc))}{k}\right], \tag{C13}$$

and we assume it to be fixed by the optimal strategy found by the coalition (in other words,  $p_e$  is not a random variable). Now, we consider different choices for the value of  $p_e$  and compute the bound in (C11) for each choice.

•  $p_e < s_l < s_{l-1}$  In this case, the expected fraction of errors is below the thresholds used in both tests. Therefore, it is likely that both tests are passed. This implies that we can trivially bound the probability of passing the test in  $P_i$ :

$$\Pr\left[T_{i,h,l}^{Doc} = 1\right] \le 1. \tag{C14}$$

On the contrary, we can use Hoeffding's inequality to bound the probability that the test in  $P_i$  fails:

$$\Pr\left[T_{j,h,l-1}^{Doc} = 0\right] = \Pr\left[\sum_{r \in R_{h \to j}} g(t_r, f_r(Doc)) \ge s_{l-1}k\right]$$

$$= \Pr\left[\sum_{r \in R_{h \to j}} g(t_r, f_r(Doc)) - p_e k \ge (s_{l-1} - p_e)k\right]$$

$$\le \exp\left(-2k(s_{l-1} - p_e)^2\right). \tag{C15}$$

By combining the bounds in (C14) and (C15) and the condition on  $p_e$ , we obtain the following upper bound on the success probability of the attack from (C11):

$$\Pr[\text{attack on } P_i, P_j] \le N(1 - d_R) \exp\left(-2k(s_{l-1} - s_l)^2\right).$$
 (C16)

•  $p_e > s_{l-1} > s_l$  In this case, the expected fraction of errors exceeds both thresholds, thus likely causing both tests to fail. We thus obtain the following bounds:

$$\Pr\left[T_{j,h,l-1}^{Doc} = 0\right] \le 1,$$

$$\Pr\left[T_{i,h,l}^{Doc} = 1\right] = \Pr\left[\sum_{r \in R_{h \to i}} g(t_r, f_r(Doc)) < s_l k\right]$$

$$= \Pr\left[p_e k - \sum_{r \in R_{h \to i}} g(t_r, f_r(Doc)) > (p_e - s_l) k\right]$$

$$\le \exp\left(-2k(p_e - s_l)^2\right),$$
(C18)

where the second bound is again obtained by applying Hoeffding's inequality. By combining the above bounds and the condition on  $p_e$ , we obtain the following upper bound on the success probability of the attack:

$$\Pr[\text{attack on } P_i, P_j] \le N(1 - d_R) \exp\left(-2k(s_{l-1} - s_l)^2\right). \tag{C19}$$

•  $s_l < p_e < s_{l-1}$  In this case, it is likely that the test in  $P_i$  fails while the test in  $P_j$  succeeds. Thus, we can use Hoeffding's inequality to bound both probabilities in (C11):

$$\Pr\left[T_{i,h,l}^{Doc} = 1\right] \le \exp\left(-2k(p_e - s_l)^2\right)$$
 (C20)

$$\Pr\left[T_{j,h,l-1}^{Doc} = 0\right] \le \exp\left(-2k(s_{l-1} - p_e)^2\right). \tag{C21}$$

By combining the above bounds in (C11), we obtain the following upper bound on the attack success probability:

$$\Pr[\text{attack on } P_i, P_j] \le N(1 - d_R) \min \left\{ \exp\left(-2k(p_e - s_l)^2\right), \exp\left(-2k(s_{l-1} - p_e)^2\right) \right\}. \tag{C22}$$

Since the optimal strategy by the coalition aims at maximizing the attack success probability, we choose  $p_e$  to be equidistant from  $s_l$  and  $s_{l-1}$ , i.e.  $p_e = (s_l + s_{l-1})/2$ . Indeed, this choice maximizes the right hand side of the last expression. We obtain:

$$\Pr[\text{attack on } P_i, P_j] \le N(1 - d_R) \exp\left(-\frac{k}{2}(s_{l-1} - s_l)^2\right).$$
 (C23)

By comparing the bounds (C16), (C19), and (C23) obtained for various possible choices of the value of  $p_e$ , we observe that the largest attack probability on a given pair of participants  $P_i$ ,  $P_j$  is obtained in (C23) for  $p_e = (s_l + s_{l-1})/2$ . Thus, we obtain the following upper bound on the probability of a successful attack on the fixed pair  $P_i$ ,  $P_j$  of honest participants, given that the coalition C performs the attack with invalid hash functions provided to honest receivers:

$$\Pr[\text{attack on } P_i, P_i] \le \varepsilon_{i,i},$$
 (C24)

where we employed the optimal choice for the spacing of the  $s_l$  variables,  $s_{l-1} - s_l = \Delta s \approx [2(l_{max} + 1)]^{-1}$ ,

$$\varepsilon_{i,j} = N(1 - d_R) \exp\left(-\frac{k}{8(l_{max} + 1)^2}\right). \tag{C25}$$

At this point, we turn to consider the attack on the transferability property between  $P_i$  and  $P_j$  which is enabled by imperfect authenticated channels, while assuming that all the hash functions distributed by the sender are correct. Specifically, the coalition can either attack the authenticated channels used to forward the  $\{Doc, Sig\}$  pair in the messaging stage, or the authenticated secret channels used to shuffle the hash functions in the distribution stage. For the former, a successful attack would indeed cause a rejection but it comes at the cost of having one of the participants verifying a different pair  $\{Doc', Sig'\}$ . Since the transferability definition requires both parties to verify the same document-signature pair, this scenario is not relevant for the proof (although causing a transferability issue in practice). In the latter attack, the coalition attempts to alter the hash functions sent to party  $P_j$  over the authenticated secret channels by honest receivers, such that  $P_j$  rejects the signature. Meanwhile, the hash functions destined to  $P_i$  are left untouched, hence the event  $\text{Ver}_{i,l}(Doc, Sig) = \text{True occurs}$  with certainty. The probability that the described attack is successful is given by:

$$\Pr\left[\text{attack on } P_{j}\right] = \Pr\left[\text{Ver}_{j,l-1}(Doc, Sig) = \text{False}\right] \\
= \Pr\left[\sum_{h=1}^{N(1-d_{R})-1} T_{j,h,l-1}^{Doc} \le \frac{N}{2} + lNd_{R} - 1\right], \tag{C26}$$

where we already accounted for the fact that the  $Nd_R$  dishonest receivers will send invalid hash functions to  $P_j$  and that the hash functions kept by  $P_j$ , i.e. those in the set  $F_{j\to j}$ , are necessarily correct, thus causing  $T_{j,i,l-1}^{Doc}=1$ .

From (C26), we deduce that  $P_j$  rejects the signature if at least  $N(1-d_R)-1-(N/2+lNd_R-1)=\tilde{N}[1/2-(l+1)d_R]$  tests are not passed. This implies that the coalition must successfully corrupt the hash functions sets  $F_{h\to j}$  sent by an equal number of honest receivers over authenticated secret channels. Now, recall from Sec. II that the authenticated channels are implemented via WC authentication with key recycling and with the hash family  $\mathcal{F}_{AXU}$ . Then, the probability of corrupting one authenticated message of length  $b_M$  is  $b_M/2^{b'_H-1}$  according to Table I. Note, however, that a single failed authentication (caused by a corruption attempt) causes the protocol to abort. Then, the probability that the coalition successfully corrupts enough hash function sets to cause  $P_j$ 's rejection without causing an abortion reads:

$$\Pr\left[\text{attack on } P_j\right] = \left(\frac{ky + k \log_2(Nk)}{2^{b_H'-1}}\right)^{N[1/2 - (l+1)d_R]}$$

$$\leq \varepsilon_{\text{auth}}, \tag{C27}$$

where  $ky + k \log_2(Nk)$  is the length of the messages exchanged by the receivers in the distribution stage and where

$$\varepsilon_{\text{auth}} := \left(\frac{ky + k \log_2(Nk)}{2^{b'_H - 1}}\right)^{N[1/2 - (l_{max} + 1)d_R]}.$$
 (C28)

Now, for each given pair  $P_i$ ,  $P_j$ , the coalition might decide to perform the first attack we analyzed, whose probability of success is given by (C25), or the second attack with probability of success (C28), or a combination of the two. Combining the two attacks means that the verification tests at  $P_j$  may fail either due to incorrect hash functions received from honest receivers, or due to hash functions sent by honest receivers which are corrupted on their way to  $P_j$ . The necessary events for the hybrid attack to take place are that  $P_i$  successfully verifies the signature and that at least one successful attack is performed on the authenticated channels. Thus, the success probability of the hybrid approach is bounded by:

$$\Pr [\text{hybrid attack}] \leq \Pr [\text{Ver}_{i,l}(Doc, Sig) = \text{True}] \frac{ky + k \log_2(Nk)}{2^{b'_H - 1}} \\
= \Pr \left[ \sum_{h=1}^{N(1-d_R)} T_{i,h,l}^{Doc} > \frac{N}{2} + lNd_R \right] \frac{ky + k \log_2(Nk)}{2^{b'_H - 1}} \\
\leq \frac{ky + k \log_2(Nk)}{2^{b'_H - 1}} \Xi \left( \left\lfloor \frac{N}{2} + lNd_R \right\rfloor + 1, N(1 - d_R), e^{-\frac{k}{2}(s_{l-1} - s_l)^2} \right) \\
\leq \varepsilon_{\text{hyb}}, \tag{C29}$$

with

$$\varepsilon_{\text{hyb}} := \frac{ky + k \log_2(Nk)}{2^{b'_H - 1}} \Xi\left(\left\lfloor \frac{N}{2} \right\rfloor + 1, N(1 - d_R), \exp\left(-\frac{k}{8(l_{max} + 1)^2}\right)\right).$$
(C30)

In the first equality we accounted for the fact that the dishonest receivers will provide  $P_i$  with correct hash functions. In the second inequality, we considered the probability of passing a single test at  $P_i$  as per derivation of (C23) and used the function (C5) for the probability of at least  $\frac{N}{2} + lNd_R$  successes out of  $N(1 - d_R)$  attempts. Finally, in the last inequality we chose the smallest value<sup>2</sup> for l and adopted the optimal spacing  $s_{l-1} - s_l = \Delta s = [2(l_{max} + 1)]^{-1}$ .

Since the coalition aims at maximizing its attack success probability, for each fixed pair  $P_i, P_j$  the coalition will perform the type of attack with the highest probability of success, which is given by:

$$\max\{\varepsilon_{i,i}, \varepsilon_{\text{auth}}, \varepsilon_{\text{hvb}}\}. \tag{C31}$$

Recall that the total success probability in (C8) refers to any pair of honest participants. There are in total  $\binom{N(1-d_R)}{2}$  different pairs of honest participants. By employing the union bound, the relevant probability can be bounded by:

$$\Pr\left[\exists P_i, P_j \notin C : \operatorname{Ver}_{i,l}(Doc, Sig) = \operatorname{True} \wedge \operatorname{Ver}_{j,l-1}(Doc, Sig) = \operatorname{False}\right] \leq \binom{N(1-d_R)}{2} \max\{\varepsilon_{i,j}, \varepsilon_{\operatorname{auth}}, \varepsilon_{\operatorname{hyb}}\}$$
(C32)

l=0 so that the resulting bound is also reusable in the proof against repudiation.

<sup>&</sup>lt;sup>2</sup> We remark that, according to the transferability definition, the smallest allowed value for l is l=1. Here, however, we choose

where the epsilon variables are given in (C25), (C28) and (C30), respectively. This concludes the proof.

**Lemma.** Protocol 3 is  $\varepsilon_{\text{rep}}$ -secure against repudiation according to Definition III.3, with:

$$\varepsilon_{\text{rep}} = {N(1 - d_R) \choose 2} \max\{\varepsilon_{i,j}, \varepsilon_{\text{auth}}, \varepsilon_{\text{hyb}}\},$$
 (C33)

where  $\varepsilon_{i,j}$ ,  $\varepsilon_{\text{auth}}$  and  $\varepsilon_{\text{hyb}}$  are given in (C25), (C28) and (C30), respectively.

*Proof.* The proof can be reduced to a special case of the proof of transferability (see proof of Lemma III.7). According to the repudiation definition (Definition III.3), we need to bound the following probability:

$$p_{\text{rep}} = \Pr\left[\exists P_i \notin C : \text{Ver}_{i,l}(Doc, Sig) = \text{True} \land \text{MV}(Doc, Sig) = \text{Invalid}\right].$$
 (C34)

Here, the attack is successful if an honest receiver validates the signature while the majority of receivers (specifically,  $\geq \lceil N/2 \rceil$ ) deems it invalid at verification level l=-1. With a coalition of  $Nd_R=\omega-1$  dishonest receivers, we can already be certain that  $\omega-1$  participants will deem the signature as invalid in the majority vote. This means that the attack is successful when at least  $N_{HR} := \lceil N/2 \rceil + 1 - \omega$  honest receivers reject the signature at verification level l=-1. By taking into account that the number of dishonest participants is strictly bounded by:  $\omega < (N+1)/2$ , the number of honest receivers that need to reject the signature for the attack to be successful is at least  $N_{HR} > \lceil N/2 \rceil - N/2 + 1/2$ , which is equivalent to requesting:

$$N_{HR} \ge \begin{cases} 2 & \text{for } N \text{ odd} \\ 1 & \text{for } N \text{ even.} \end{cases}$$
 (C35)

In other words, a precondition for the attack being successful for any N is that at least one honest receiver rejects the signature at verification level l = -1. Then, the probability to be bounded reads:

$$p_{\text{rep}} \leq [\exists P_i, P_j \notin C : \text{Ver}_{i,l}(Doc, Sig) = \text{True} \land \text{Ver}_{j,-1}(Doc, Sig) = \text{False}].$$
 (C36)

Now, we use the fact pointed out in (18) to upper bound the last expression as follows:

$$p_{\text{rep}} \leq [\exists P_i, P_j \notin C : \text{Ver}_{i,0}(Doc, Sig) = \text{True} \land \text{Ver}_{j,-1}(Doc, Sig) = \text{False}]$$

$$\leq \binom{N(1-d_R)}{2} \max\{\varepsilon_{i,j}, \varepsilon_{\text{auth}}, \varepsilon_{\text{hyb}}\}, \tag{C37}$$

where we used (C32) in the last inequality, since it also valid for the special case l=0. This concludes the proof.  $\Box$ 

**Lemma.** Protocol 3 is  $\varepsilon_{\text{for}}$ -secure against forgery according to Definition III.1, with:

$$\varepsilon_{\text{for}} = (N - \omega) \Xi(|N/2|, N - \omega, p_t),$$
 (C38)

where  $p_t$  is defined as:

$$p_t = \Xi(\lfloor k(1 - s_0) \rfloor + 1, k, 2^{1 - b_H}),$$
 (C39)

and

$$\Xi(k, n, p) := \sum_{j=k}^{n} \binom{n}{j} p^{j} (1-p)^{n-j}.$$
 (C40)

Moreover, the forgery attack based on the security loophole discussed in Ref. [15], where a dishonest receiver forges the pair  $\{Doc', Sig'\}$  and invokes the dispute resolution method for the other parties to accept the pair, succeeds with probability:

$$p_{\text{attack}} = \Xi(|N/2| + 1 - \omega, N - \omega, p_{-1}),$$
 (C41)

with:

$$p_{-1} = \Xi(|N/2| + 1 - \omega, N - \omega, p_t). \tag{C42}$$

*Proof.* A successful forgery attack occurs when a coalition C of  $\omega$  dishonest receivers, which does not include the sender, is provided with a valid  $\{Doc, Sig\}$  pair and finds a new pair  $\{Doc', Sig'\}$  (with  $Doc' \neq Doc$ ) such that at least one of the  $N - \omega$  honest receivers accepts it at the lowest verification level, l = 0.

We first consider the case where the coalition tries to deceive a fixed receiver,  $P_i$ . Then, we are interested in bounding the following probability:

$$\Pr\left[\operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] = \Pr\left[\sum_{j=1}^{N} T_{i,j,0}^{Doc'} \ge \lfloor N\delta_0 \rfloor + 1\right]. \tag{C43}$$

By using the fact that  $\delta_0 = 1/2 + d_R$  and that  $d_R = (\omega - 1)/N$ , we can write:

$$\Pr\left[\operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] = \Pr\left[\sum_{j=1}^{N} T_{i,j,0}^{Doc'} \ge \left\lfloor \frac{N}{2} \right\rfloor + \omega\right]. \tag{C44}$$

Now, for all the  $P_j \in C$ , the test  $T_{i,j,0}^{Doc'}$  can be forced to pass. Indeed, the coalition knows the hash functions in  $F_{j\to i}$  and thus can choose the tags contained in Sig', corresponding to the positions  $R_{j\to i}$ , to be the hashed values of Doc' when using the functions in  $F_{j\to i}$ . This will guarantee that  $T_{i,j,0}^{Doc'}=1$  occurs with certainty. Then, we can update the probability in (C44) as follows:

$$\Pr\left[\operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] = \Pr\left[\sum_{h=1}^{N-\omega} T_{i,h,0}^{Doc'} \ge \left\lfloor \frac{N}{2} \right\rfloor\right],\tag{C45}$$

meaning that the attack is successful if at least  $\lfloor N/2 \rfloor$  additional tests originating from honest receivers  $h \notin C$  are passed. Let  $p_t$  be the probability that the test based on the hash functions sent by receiver h is passed:

$$p_t := \Pr\left[T_{i,h,0}^{Doc'} = 1\right]. \tag{C46}$$

Then, assuming w.l.o.g. that the probability of passing each test is independent for different receivers h, we can express the probability in (C45) as follows:

$$\Pr\left[\operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] = \Xi(|N/2|, N - \omega, p_t), \tag{C47}$$

where  $\Xi(k, n, p)$ , defined in (C40), is the function returning the probability of at least k successes out of n trials with success probability p for each trial.

We now consider the general case of the coalition trying to deceive at least one of the  $N-\omega$  honest receivers. The relevant probability is thus:

$$\Pr\left[\exists P_i \notin C : \operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] \le (N - \omega) \Pr\left[\operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right], \tag{C48}$$

where we employed the union bound in the inequality. Then, by combining (C48) with (C47) we obtain:

$$\Pr\left[\exists P_i \notin C : \operatorname{Ver}_{i,0}(Doc', Sig') = \operatorname{True}\right] \le (N - \omega) \ \Xi(|N/2|, N - \omega, p_t), \tag{C49}$$

which is the claim of the Lemma.

We now derive an explicit expression for  $p_t$ . Participant  $P_i$  deems the test  $T_{i,h,0}^{Doc'}$  as passed (at verification level l=0) if the number of mismatches between the tags in Sig' and the hashes obtained by applying the functions in  $F_{h\to i}$  to Doc' is smaller than  $ks_0$ :

$$\Pr\left[T_{i,h,0}^{Doc'} = 1\right] = \Pr\left[\sum_{r \in R_{h \to i}} g(t_r', f_r(Doc')) < ks_0\right]. \tag{C50}$$

In order for the coalition to append correct tags in Sig', they need to know the hash functions in  $F_{h\to i}$ . However, this is not possible since h is an honest receiver and the sender is not part of the coalition. Let  $f_1, f_2, \ldots, f_k$  be the k hash functions in  $F_{h\to i}$ . For each hash function, the coalition knows a valid message-tag pair from the knowledge of  $\{Doc, Sig\}$  and wishes to find another valid pair. In particular, the coalition knows that Doc and  $t_1$  are such that  $f_1(Doc) = t_1$  and wants to find  $t'_1$  such that  $f_1(Doc') = t'_1$ . Since the  $\varepsilon$ -ASU<sub>2</sub> family  $\mathcal{F}_{ASU}$  adopted by the protocol has security parameter  $\varepsilon = 2^{1-b_H}$  (see Appendix B), the probability that the coalition correctly guesses the tag  $t'_1$  is given

by  $2^{1-b_H}$ . Since correctly guessing the tag for each hash function in  $F_{h\to i}$  are independent events, the probability of correctly guessing more than  $k-ks_0$  tags out of k attempts is:

$$p_t = \Pr\left[T_{i,h,0}^{Doc'} = 1\right] = \Xi(\lfloor k(1 - s_0) \rfloor + 1, k, 2^{1 - b_H}),$$
 (C51)

which is the expression provided in the claim of the Lemma.

Finally, we compute the success probability  $(p_{\text{attack}})$  of a forgery attempt where the forger invokes the dispute resolution method by falsely claiming that they verified the pair  $\{Doc', Sig'\}$  at level l=0 (this attack was first pointed out in Ref. [15]). In this case, the forger aims at making other honest receivers accept the signature at level l=-1, i.e., at a lower level compared to the standard definition of security against forgery, such that the outcome of the dispute resolution is to accept the pair: MV(Doc', Sig') = Valid.

Let  $P_i \notin C$  be an honest receiver. Then, the probability that  $P_i$  accepts the forged signature is:

$$p_{-1} := \Pr\left[\operatorname{Ver}_{i,-1}(Doc', Sig') = \operatorname{True}\right]$$

$$= \Pr\left[\sum_{j=1}^{N} T_{i,j,-1}^{Doc'} \ge \lfloor N\delta_{-1} \rfloor + 1\right]$$

$$= \Pr\left[\sum_{h=1}^{N-\omega} T_{i,h,-1}^{Doc'} \ge \left\lfloor \frac{N}{2} \right\rfloor + 1 - \omega\right], \tag{C52}$$

where we assumed that the dishonest receivers will automatically force the test to be passed  $(T_{i,j,-1}^{Doc'}=1)$ . By again using the fact that  $p_t$  is the probability that a single test  $T_{i,h,-1}^{Doc'}$  is passed, we have the following probability that an honest receiver accepts the forged signature at level l=-1:

$$p_{-1} = \Xi(|N/2| + 1 - \omega, N - \omega, p_t). \tag{C53}$$

Now consider that, to enforce MV(Doc', Sig') = Valid, the malicious coalition needs a total of  $\lfloor N/2 \rfloor + 1$  receivers accepting the pair. Since  $\omega$  of them will surely accept, only  $\lfloor N/2 \rfloor + 1 - \omega$  honest receivers out of  $N - \omega$  need to accept the pair. This occurs with probability:

$$p_{\text{attack}} = \Xi(|N/2| + 1 - \omega, N - \omega, p_{-1}),$$
 (C54)

as reported in the claim of the Lemma. This concludes the proof.