From Permissioned to Proof-of-Stake Consensus

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Abstract

This paper presents the first generic compiler that transforms any permissioned consensus protocol into a proof-of-stake permissionless consensus protocol. For each of the following properties, if the initial permissioned protocol satisfies that property in the partially synchronous setting, the consequent proof-of-stake protocol also satisfies that property in the partially synchronous and quasi-permissionless setting (with the same fault-tolerance): consistency; liveness; optimistic responsiveness; every composable log-specific property; and message complexity of a given order. Moreover, our transformation ensures that the output protocol satisfies accountability (identifying culprits in the event of a consistency violation), whether or not the original permissioned protocol satisfied it.

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1 Introduction

1.1 Permissioned and Permissionless Consensus

Blockchain protocols are run by large collections of processes that must stay in sync about the current state of the protocol's virtual machine. Keeping a distributed network of processes in agreement in the face of failures, attacks, and a potentially unreliable communication network—the problem of *consensus*—is a hard problem, but one for which there is a large body of research developed over the past four-plus decades. And indeed, many major blockchain protocols achieve their guarantees by building on protocols and ideas developed in the 1980s and 1990s.

Beyond standing on the shoulders of giants, the rise of Internet-scale blockchain protocols has pushed the state-of-the-art of consensus protocols significantly, in two distinct directions. First, 20th-century consensus protocols were typically designed for the *permissioned* setting, in which the protocol is to be run by an a priori known and fixed set of processes (e.g., a bunch of dedicated servers bought by a corporation for that purpose). Blockchain protocols, starting with the Bitcoin protocol, typically aspire to run in the *permissionless* setting in which there is free entry and exit to the set of processes running the protocol (perhaps after the acquisition of a costly resource, such as the protocol's native cryptocurrency). Permissionless consensus is strictly harder than permissioned consensus, due to a combination of additional

challenges, including sybil-resistance (most commonly addressed through proof-of-work or proof-of-stake), an ever-changing set of processes running the protocol, and the possibility of non-faulty processes periodically going offline.

Second, even after setting aside the challenges posed by the permissionless setting, the practical deployment of and competition between Internet-scale state machine replication protocols have fueled innovation in the design of permissioned consensus protocols. One good example is the recent rise of DAG-based consensus protocols, which differ from previous protocols in, among other things, their use of simultaneous block proposals by all validators to overcome the bottlenecks typical of "leader-based" protocols in the lineage of PBFT [24]. While the formal analysis of DAG-based protocols has been confined to the permissioned setting thus far [51, 80, 79, 8, 34], Sui is an example of a permissionless proof-of-stake (PoS) system using a DAG-based protocol in production (cf. [9]).

1.2 The Dream: A Generic Property-Preserving Compiler

This paper pursues the "automatic tech transfer" of innovations for permissioned consensus protocols to those for permissionless protocols. The holy grail would be a "compiler" that takes as input an arbitrary permissioned protocol and outputs an "equally good" permissionless version. Ideally, such a compiler would obviate the need for any bespoke work to extend the *design* of a permissioned protocol into a permissionless one, and similarly for its *analysis*. Given that we make no assumptions about how the initial permissioned protocol works—i.e., it is given only as a "black box"—such a compiler can only interact with the protocol by executing it directly. Similarly, the analysis of the compiler's output would be able to rely only on the fact that the initial permissioned protocol satisfies the desired properties and not, for example, on any particular design or analysis method by which those properties might be achieved.

Compromise #1: Restriction to the quasi-permissionless setting. The main result of this paper is indeed a "compiler" of the above type, but certain compromises are, or appear to be, unavoidable. First, "sufficiently permissionless" protocols inherently suffer from provable limitations that are not shared by permissioned protocols, and for this reason we focus on the quasi-permissionless setting. In more detail, Lewis-Pye and Roughgarden [64] classify the "permissionlessness of a protocol"—or more accurately, the maximum permissionlessness under which a protocol functions as intended—via a hierarchy with several levels. The permissioned setting is one extreme point of the hierarchy. The next-most restrictive setting is the *quasi-permissionless (QP)* setting; the assumption here, when specialized to proof-of-stake protocols, is that correct processes with a positive amount of locked-up stake are always active. (But unlike the permissioned setting, the set of such processes can be ever-changing, and correct processes without locked-up stake can be periodically inactive.) The next level in the hierarchy is the *dynamically available (DA)* setting in which, similar to the sleepy setting of [74], correct processes (including those with locked-up stake) may be periodically offline. Protocols that have even minimal consistency and liveness guarantees in the DA setting cannot, for example, be consistent under asynchrony, or accountable (even in synchrony), or optimistically responsive (even in synchrony) [70, 64, 69]. If the assumptions on the setting are strengthened from DA to QP, all of the properties are achievable [64]. As we are interested in a compiler that preserves properties like consistency and responsiveness, we confine our analysis of permissionless protocols to the QP setting.

Compromise #2: Property-specific analysis. The strongest-possible assertion that the compiler's (permissionless) output protocol shares all the desired properties of its

(permissioned) input protocol would be some type of homomorphism between executions (in the spirit of e.g. [31]). Such a homomorphism would map an execution of the permissionless protocol to one or more executions of the permissioned protocol showing that, intuitively, whatever could go wrong in a permissionless execution could have already gone wrong in a permissioned execution. There are several seemingly insurmountable obstacles to achieving this goal. To spell out one just example, such a homomorphism would presumably establish a relationship between the units of participation in the permissionless protocol (e.g., staked coins in a PoS protocol) and those in the original permissioned protocol (e.g., individual processes). But in an execution of the permissionless protocol, coins might well be transferred back and forth between correct and Byzantine processes (even if "locked" for certain durations of the execution), which would seem to translate to an execution of the permissioned protocol with a mobile adversary. Such a homomorphism cannot hope to preserve any property that is achievable with a static adversary but unachievable with a mobile adversary.

Because of the seeming impossibility of a property-preserving mapping between executions of the permissioned and permissionless protocols, we instead prove that our compiler preserves (a long list of) specific properties using direct, property-specific arguments.

Compromise #3: Restrictions on the preservable properties. Because the set of processes in a permissioned protocol is fixed while that of a permissionless protocol must update periodically (e.g., to reflect changes in processes' stakes), and because the permissioned protocol is given only as a closed box, any implementation of the desired compiler must presumably execute the permissioned protocol repeatedly, updating the process set appropriately each time. We refer to each execution of the permissioned protocol as an *epoch*, and this approach to the compiler's design as *epoch-based*. The naive hope would then be that the assumed properties of the permissioned protocol hold in each epoch of the permissionless protocol's execution, and therefore also for the overall execution of that protocol.

Neither part of this hope can be carried out without restrictions on the properties we aim to preserve. The first part of the hope breaks down for, for example, properties of the form "eventually, the execution satisfies some predicate φ." The issue here is that while every (infinite) execution of a permissioned protocol might eventually satisfy φ, there may be no finite epoch length for which subexecutions of the protocol are guaranteed to satisfy φ. The second part of the hope breaks down for properties that are not preserved by the "concatenation" of the executions of two consecutive epochs. For a trivial example, the property that "at most n distinct identifiers ever vote" will be true for a PBFT-style permissioned protocol with a fixed set of n processes, but will not generally be true for any corresponding permissionless protocol (for which there is an unbounded number of identifiers, any of which may acquire stake and vote at some point in an execution).

The hope, then, is that an epoch-based compiler might still be capable of preserving all of the specific properties that one would be interested in preserving. This brings us to the main result of the paper.

1.3 The Main Result

The primary contribution of this paper is a generic compiler from permissioned to proof-of-stake permissionless protocols in the quasi-permissionless and partially synchronous setting. The compilation process is well defined for any permissioned protocol. For each of the following properties, if the initial permissioned protocol satisfies that property (in the partially synchronous setting), then the consequent proof-of-stake protocol also satisfies that property

(in the partially synchronous and quasi-permissionless setting, and with the same fault-tolerance): consistency; liveness with respect to a liveness (latency) parameter ℓ;[1] optimistic responsiveness; and message complexity of a given order. Furthermore, our construction preserves all composable log-specific safety properties—that is, properties defined solely over logs (the outputs of processes; cf. Sec. 2) and preserved under concatenation (cf. the discussion in the "Compromise #3" paragraph above): roughly speaking, if two consistent logs L_1 and L_2 each satisfy such a property, then their concatenation $L_1 \| L_2$ also satisfies it. Finally, our compiler guarantees that the output protocol satisfies accountability [64, 29, 49, 30, 77, 70, 71]—that is, in the event of a consistency violation, the responsible parties can be correctly identified—regardless of whether the original permissioned protocol possessed this property. For example, applying our transformation to existing permissioned DAG-based protocols (e.g., [51, 80, 79, 8, 34]) yields new quasi-permissionless DAG-based protocols that had not previously been formally analyzed or proven correct.[2]

1.4 Why a Generic Transformation Was Needed

Limitations of existing practical approaches. Several blockchain systems in practice—such as Sui, Aptos, Cosmos, and Celo—already employ partially synchronous, quasi-permissionless consensus protocols. These systems typically adapt or build upon well-known permissioned protocols like HotStuff [86] (in the case of Aptos and Celo), Tendermint [18] (in the case of Cosmos), or Mysticeti [9] (in the case of Sui). While these protocols achieve significant practical performance and are deployed at scale, they rely on *protocol-specific modifications* and often embed *ad-hoc reconfiguration mechanisms* to work in PoS/QP settings. In particular, they are designed and analyzed in a bespoke manner—each new protocol or setting requires revisiting core design choices, reestablishing safety and liveness guarantees, and adapting reconfiguration logic to ensure correctness. To date, there exists *no general-purpose method* for converting an arbitrary permissioned protocol into a quasi-permissionless PoS protocol with well-defined theoretical guarantees.

Theoretical efforts & their shortcomings. Prior theoretical work has investigated how to adapt specific permissioned consensus protocols to PoS or quasi-permissionless settings. However, these approaches, like their practical counterparts, are fundamentally *protocol-specific* and rely on tightly coupled mechanisms that are not general. For example, Lewis-Pye and Roughgarden [64] construct a quasi-permissionless, PoS version of HotStuff [86] by embedding specialized reconfiguration mechanisms directly into the protocol. Budish *et al.* [19] similarly transform Tendermint [18] into a quasi-permissionless, PoS protocol, but again this requires bespoke changes tailored to that specific protocol. Sui Lutris [15] is yet another protocol-specific design, featuring a custom hybrid architecture with partial and total ordering, and an ad-hoc epoch-based reconfiguration mechanism. Other systems, such as Hybrid Consensus [73] and PaLa [25], proceed in epochs and incorporate reconfiguration, but are built from the ground up with specific assumptions and analysis for a given protocol architecture. More broadly, reconfiguration—the task of updating the set of validators—has been widely studied [40, 39, 72, 13], but existing solutions embed reconfiguration logic deeply within the protocol's internal machinery. None of these approaches offer a generic, reusable

[1] The liveness parameter is passed as input to the compiler and the epoch length of the output protocol will depend on its value (cf. Sec. 4).

[2] To the best of our knowledge, Sui Lutris [15] is the only DAG-based protocol that has been formally analyzed in the quasi-permissionless setting, explicitly addressing the challenge of validator changes through a reconfiguration mechanism.

method for reconfiguration mechanisms or for achieving quasi-permissionless PoS protocols.

Why a generic transformation is needed. This disconnect—between *theory*, which mainly focuses on permissioned consensus, and *practice*, which needs permissionless consensus—is precisely what motivates our work. We present the first *generic, closed-box transformation* that converts any partially synchronous permissioned protocol into a quasi-permissionless PoS protocol, while preserving a rich set of properties: consistency, liveness, optimistic responsiveness, accountability, and all composable log-specific safety properties. Our transformation separates the consensus core from reconfiguration logic, enabling a clean and composable analysis. This modularity provides two key benefits:

- For theorists, it allows continued focus on designing and analyzing permissioned protocols, while ensuring their results can be lifted automatically to the permissionless world.
- For practitioners, it offers a principled and reusable path to deploying proven permissioned protocols in permissionless environments, without redesigning reconfiguration or re-establishing correctness from scratch.

Essentially, our transformation acts as a "bridge lemma" between permissioned and permissionless worlds, making decades of theoretical work directly applicable to modern PoS systems.

Why designing this transformation was challenging. Naively running an arbitrary permissioned protocol as a subroutine within a PoS protocol—restarting it periodically with a new set of processes reflecting the latest stake amounts—fails to preserve even the most basic properties. Next, we highlight key technical challenges in designing and analyzing our generic compiler, along with the ideas that contribute to our solution.

Quit-enhanced permissioned protocols. At a high level, our compiler follows an epoch-based approach, executing the given permissioned protocol in each epoch for a finite duration. Therefore, the first step in our approach is to analyze, in a general manner, the behavior of permissioned protocols when processes are allowed to quit executing the protocol (which corresponds to the end of an epoch). The challenge here is the fact that we know nothing about how the given permissioned protocol might work, and the worry is that interfering with its execution (e.g., making one process inactive so that a different process can take its place) could affect its properties in unpredictable ways (e.g., with the newly inactive process now viewed as Byzantine by the protocol, violating its assumed fault-tolerance). For instance, consider adapting the Tendermint [18] permissioned consensus protocol to a quasi-permissionless setting using our epoch-based approach. Recall that Tendermint, being a permissioned protocol, assumes that all correct processes participate in the protocol forever, i.e., they never quit executing the protocol. As a result, all of its proven guarantees are built on this crucial assumption. However, when we attempt to transfer Tendermint into a quasi-permissionless setting using our epoch-based approach, this assumption no longer holds. Specifically, if the transition from epoch e to epoch $e + 1$ takes place before the network has stabilized (i.e., before GST; cf. Sec. 2 for the definition of GST), it is possible that all correct processes except one stop participating in the Tendermint instance associated with epoch e. Therefore, the remaining correct process finds itself isolated, surrounded only by adversarial processes, with no support from other correct participants. This leads to a critical question: can consistency still be preserved in epoch e under such circumstances? In particular, is there a risk that the abandoned correct process could be misled by adversarial processes into violating consistency? Notably, this case is *not* covered by the original Tendermint analysis, as it falls outside the boundaries of the permissioned model assumed in [18], where such an abandoned correct process is never considered. Thus, analyzing the consistency (and other properties) of the resulting quasi-permissionless protocol requires going beyond the

guarantees provided by the original Tendermint permissioned protocol, as those guarantees no longer directly apply in this new setting.

To address this challenge, we proceed as follows: (1) we extend the interface of Tendermint—though the idea applies to any permissioned protocol—to allow correct processes to stop participating, and (2) we analyze the guarantees provided by this extended protocol. We refer to these enriched versions of permissioned protocols as *quit-enhanced* permissioned protocols. Given any standard permissioned protocol, in which correct processes are expected to participate forever, the quit-enhanced version behaves identically in terms of internal logic but explicitly permits correct processes to stop executing the protocol. Thus, quit-enhancing a standard permissioned protocol constitutes a closed-box transformation: only the interface is extended, while the internal mechanisms remain unchanged (and untouched). Importantly, quit-enhanced permissioned protocols integrate naturally with our epoch-based structure: the properties of quit-enhanced permissioned protocols extend directly to our setting. With this in mind, we prove that quit-enhanced versions of arbitrary permissioned protocols preserve the key properties of interest (such as consistency and liveness) originally established in the standard permissioned model, and are thus suitable for use within an epoch-based compiler.

Preserving consistency. Even the basic property of consistency will not be preserved in an epoch-based approach to a generic compiler without carefully designing how one epoch transitions into the next. For example, consider a permissioned protocol whose algorithm first tentatively confirms a block—such as upon receiving an initial quorum certificate—and only later finalizes it, either through a follow-up quorum certificate or because the block is extended by blocks that become finalized. (It is important to emphasize that this tentative confirmation is an internal step within the algorithm's operation and does not represent the block's external status.) If an epoch-ending block B is only tentatively confirmed, who is then allowed to extend that block? Which processes should constitute the new validator set? We cannot allow the validator set for the next epoch to be determined based on the transactions in block B and its ancestors, with new validators immediately proposing descendants of B, because B has not been finalized. Since another conflicting block B' might also be tentatively confirmed, this could result in differing views on which validator set should be chosen for the next epoch. If we simply wait for the first directly finalized block and treat it as the genesis block of the next epoch, a key question remains: How can we unambiguously identify which block was directly finalized first?

To overcome this challenge, we introduce the notion of an *epoch-ending block* (akin to epoch transition in [73]). When the validators of an epoch e determine that the time has come to end the epoch (the conditions for this are discussed in the "Preserving liveness" paragraph below), they issue special epoch-ending transactions. They then wait for the first finalized block that, along with its ancestors, finalizes epoch-ending transactions from at least a quorum of the epoch's validators. This uniquely determined block becomes the epoch-ending block and serves as the genesis block for the next epoch. (Any data necessary to prove finality, such as blocks that were produced after the epoch-ending one, are retained as part of the protocol's history but do not contribute to the set of transactions finalized in that epoch.)

Preserving liveness. Preserving liveness (and optimistic responsiveness, a strengthening of liveness) presents an orthogonal set of challenges. For example, in the partially synchronous model, epoch lengths are most sensibly denominated in blocks (rather than time). The assumed liveness parameter ℓ of the underlying permissioned protocol—stating that newly issued transactions are finalized within ℓ time after the network stabilizes—is, however, defined in terms of time steps. Naturally, this is a desirable property that we aim to preserve.

The issue is then that, if epochs complete in fewer than ℓ timesteps (due to an unexpectedly fast network) and every epoch has a fresh set of new validators, there is no guarantee that a given transaction will ever be finalized.

Our compiler addresses this challenge by ensuring that all correct validators overlap in each epoch (after the network stabilizes) for at least ℓ time, which allows for all new transactions to be finalized. To enforce this, each validator of an epoch e locally measures a period of $\ell + \Delta$ time, where Δ denotes the known bound on message delays after the network stabilizes (i.e., after GST; cf. Sec. 2). Given that messages propagate within Δ time, this local timing ensures that correct validators overlap for at least ℓ time during the epoch e (assuming the epoch takes place after stabilization). After this $\ell + \Delta$ interval elapses, a validator knows the epoch has run long enough and issues an epoch-ending transaction (as previously explained in the "Preserving consistency" paragraph). Because each epoch concludes with the finalization of an epoch-ending block—one that includes epoch-ending transactions from a quorum of validators, and thus from at least one correct validator—it is guaranteed that every epoch following network stabilization runs long enough to ensure finalization of new transactions, in accordance with the ℓ-liveness property of the underlying permissioned protocol.

Preserving composable log-specific safety properties. As discussed earlier, the epoch-based approach of our compiler necessitates focusing on properties whose satisfaction in every finite prefix of an (infinite) execution guarantees satisfaction in the entire execution—these are known as *safety* properties [5]. Consistency and optimistic responsiveness are examples of such safety properties. In contrast, eventual liveness is not a safety property, though liveness with respect to a fixed time bound ℓ on time-to-finality is.

This raises the question: how broad is the class of safety properties we can hope to preserve? Given the epoch-based approach, we must limit ourselves not only to safety properties but to those that are "closed under concatenation". To formalize this, we focus on *log-specific* properties—predicates that depend solely on the validators' running logs of finalized transactions (and not on, say, the precise sequence of messages that led to the creation of those logs). For log-specific properties, "safety" then means that a violation of the property must be evident from a finite-length prefix of (possibly unbounded-length) logs, and "composable" means that the property is preserved under unions of sets of logs. A canonical example of a composable log-specific safety property is an external validity property, such as "every finalized transaction is accompanied by appropriate signatures". We prove that our transformation preserves, simultaneously, every composable log-specific safety property.

2 System Model: Overview

In this section, we provide an overview of the system model. A detailed description can be found in App. A.

Processes, identifiers & adversary. We consider a (potentially infinite) set of processes denoted by Π. Each process $p \in \Pi$ is assigned a non-empty and potentially infinite set of *identifiers*, denoted by $\mathsf{id}(p)$. Intuitively, $\mathsf{id}(p)$ determines the set of public keys for which process p knows the corresponding private key. We denote by IDs the set of all identifiers. Moreover, each process may or may not be *active* at each timeslot. A *process allocation* is a function specifying, for each process $p \in \Pi$, the timeslots at which process p is active. To accommodate for clock drifts, our model permits processes to be idle even at timeslots at which they are active. Concretely, at each timeslot at which a process is active, the process can either be *waiting* or *not waiting*. Whether a process is waiting or not at a specific timeslot

is also determined by the process allocation function. In this work, we focus on protocols assuming a public key infrastructure (PKI) that allows processes to sign their messages and verify messages received from other processes.[3] Finally, we assume a static adversary that corrupts a fraction of all processes at the beginning of each execution. A corrupted process is said to be *faulty*; a non-faulty process is said to be *correct*.

Environment. There exists an *environment* that sends *transactions* to active and non-waiting processes. If the environment sends a transaction to an active and non-waiting process p at a timeslot τ, then p receives the transaction at the timeslot τ (along with messages sent by other processes).

Partial synchrony. This work focuses on the standard partially synchronous model [41]. In a nutshell, there exists an unknown timeslot GST such that (1) the system behaves asynchronously before GST, and (2) the system behaves synchronously after GST with the known upper-bound Δ on message delays. Moreover, if any correct process p is active at any timeslot $\tau \geq \text{GST}$, then p is not waiting at timeslot τ, i.e., no clock drift occurs after GST. Lastly, each execution is associated with an unknown duration $\delta \leq \Delta$ that denotes the *actual* bound on message delays.

Logs & stake. A *log* is a non-empty ordered list of transactions. Given any log $\mathcal{L}$, the following methods are defined:

- $\mathcal{L}.\mathsf{length}$: the number of transactions in $\mathcal{L}$.
- $\mathcal{L}[i]$, for any $i \in [1, \mathcal{L}.\mathsf{length}]$: the i-th transaction of $\mathcal{L}$.

Two logs $\mathcal{L}_1$ and $\mathcal{L}_2$ are *consistent* if and only if $\mathcal{L}_1[i] = \mathcal{L}_2[i]$, for every i with $1 \leq i \leq \min(\mathcal{L}_1.\mathsf{length}, \mathcal{L}_2.\mathsf{length})$. Otherwise, the logs are *inconsistent*. Similarly, a log $\mathcal{L}_2$ *extends* a log $\mathcal{L}_1$ if and only if (1) logs $\mathcal{L}_1$ and $\mathcal{L}_2$ are consistent, and (2) $\mathcal{L}_1.\mathsf{length} \leq \mathcal{L}_2.\mathsf{length}$. If a transaction tr belongs to a log $\mathcal{L}$, we write "$\mathsf{tr} \in \mathcal{L}$". We denote by Logs the set of all logs.

Genesis & local log. Each execution is associated with a unique *genesis log* known to all processes. The genesis log generalizes the concept of a "genesis block" and acts as the initial log from which all subsequent logs are built.

Each process maintains its *local log*. Formally, each process p has a special log-register denoted by $\mathsf{log}(p)$. For every correct process p, $\mathsf{log}(p) = \mathcal{L}_\text{g}$ at timeslot 0, where $\mathcal{L}_\text{g}$ denotes the unique genesis log (of that specific execution). Given any correct process p and any timeslot τ, $\mathsf{log}(p, \tau)$ denotes the value in the $\mathsf{log}(p)$ register at timeslot τ. If $\mathsf{log}(p, \tau) = \mathcal{L}$, we say that p *outputs* $\mathcal{L}$ at timeslot τ.

Stake. Every log defines its *stake distribution*. Formally, there exists a function $\mathsf{S} : \mathsf{Logs} \times \mathsf{IDs} \to \mathbb{N}_{\geq 0}$. Intuitively, stake refers to each identifier's amount of on-chain resources. For each log $\mathcal{L} \in \mathsf{Logs}$, we define its *total stake*:

$$\mathcal{L}.\mathsf{total\_stake} = \sum_{id \in \mathsf{IDs}} \mathsf{S}(\mathcal{L}, id).$$

We assume that, for each log $\mathcal{L} \in \mathsf{Logs}$, $\mathcal{L}.\mathsf{total\_stake} > 0$. Moreover, we set the following restriction on the considered stake function $\mathsf{S}(\cdot, \cdot)$:

$$\forall (\mathcal{L}_1, \mathcal{L}_2) \in \mathsf{Logs}^2 : \mathcal{L}_1.\mathsf{total\_stake} = \mathcal{L}_2.\mathsf{total\_stake}.$$

Given this restriction, let $\mathbb{T}$ denote the total stake, i.e., $\mathbb{T} = \mathcal{L}.\mathsf{total\_stake}$, for every $\mathcal{L} \in \mathsf{Logs}$. Importantly, we require protocols to be agnostic to the stake distribution function: given

[3] We discuss how to extend our results in Sec. 5.

any stake distribution function $\mathsf{S}(\cdot,\cdot)$ satisfying the conditions above, the protocol must meet its specification to be deemed correct.

Permissioned setting. Here, the set of processes Π is finite and known. Additionally, Π's cardinality is known. Moreover, each process has a single identifier: $\forall p \in \Pi : \mathsf{id}(p) = \{p\}$. Finally, each process is active at every timeslot.

Static ρ-bounded adversary. A static ρ-bounded adversary, for any $\rho \in [0,1]$, corruptes at most $\rho \cdot n$ processes at the beginning of each execution.

Known vs. unknown facts. The following facts are known to processes:
- the set of processes Π, its cardinality, and the identifier function $\mathsf{id}(\cdot)$;
- the bound on the power of the adversary ρ, the stake distribution function $\mathsf{S}(\cdot,\cdot)$, the genesis log, and the upper-bound Δ on message delays;
- the process allocation function as every process is active at every timeslot.

In contrast, the following facts are unknown to processes:
- the set of corrupted processes, its cardinality, and GST.

Quasi-permissionless setting. In the quasi-permissionless setting, the set of processes Π is not necessarily finite. Moreover, processes might have more than a single associated identifier. In the quasi-permissionless setting, only processes with non-zero stake are guaranteed to be active. Specifically, for any timeslot τ and any correct process p for which there exist a τ-active correct process q and an identifier $id_p \in \mathsf{id}(p)$ with $\mathsf{S}(\mathsf{log}(q,\tau), id_p) > 0$, process p is active at τ.

Static ρ-bounded adversary. Intuitively, a static ρ-bounded adversary cannot control more than a ρ fraction of the total stake. Formally, for every correct process p and every timeslot τ, at most ρ fraction of $\mathsf{log}(p,\tau)$'s total stake (i.e., $\mathsf{log}(p,\tau).\mathsf{total\_stake} = \mathbb{T}$) belongs to identifiers associated with faulty processes according to the stake distribution specified by the log $\mathsf{log}(p,\tau)$.

Known vs. unknown facts. The following facts are known to processes:
- the bound on the power of the adversary ρ, the stake distribution function $\mathsf{S}(\cdot,\cdot)$, and the genesis log;
- the upper-bound on message delays Δ.

The following facts are unknown:
- the set of processes Π, its cardinality, and the identifier function $\mathsf{id}(\cdot)$;
- the set of corrupted processes, its cardinality, and GST;
- the process allocation function.

3 Consensus Properties

This section outlines the consensus properties we aim to translate from the permissioned to the quasi-permissionless setting. These properties are divided into two categories: (1) core properties (Sec. 3.1), including consistency, liveness, optimistic responsiveness, and accountability, and (2) composable log-specific safety properties (Sec. 3.2).

3.1 Core Properties

We begin by defining the *core properties* of (permissioned or quasi-permissionless) consensus protocols, which are found in (almost) all of them.

Consistency. Intuitively, consistency guarantees that logs of correct processes never diverge, i.e., there are no "forks".

▶ **Definition 1** (Consistency). *A (permissioned or quasi-permissionless) protocol satisfies* consistency *if and only if the following two conditions hold:*

- *No roll-backs: For every correct process $p \in \Pi$ and every two timeslots $\tau_1, \tau_2 \in \mathbb{N}_{\geq 0}$ with $\tau_1 < \tau_2$, $\mathsf{log}(p, \tau_2)$ extends $\mathsf{log}(p, \tau_1)$.*
- *No divergence: For every pair of correct processes $(p_1, p_2) \in \Pi^2$ and every timeslot $\tau \in \mathbb{N}_{\geq 0}$, logs $\mathsf{log}(p_1, \tau)$ and $\mathsf{log}(p_2, \tau)$ are consistent.*

If a protocol satisfies the consistency property against a ρ-bounded static adversary, we say the protocol is *ρ-consistent*.

Liveness. The liveness property ensures that every transaction is finalized within a known time frame after GST.

▶ **Definition 2** (ℓ-Liveness). *A (permissioned or quasi-permissionless) protocol satisfies* ℓ-liveness *if and only if the following condition is satisfied for every timeslot $\tau \in \mathbb{N}_{\geq 1}$. Suppose the following holds:*

- *Let $\tau^* = \max(\tau, GST) + \ell$.*
- *Let a transaction tr be received by a correct process from the environment at some timeslot $\leq \tau$.*
- *Let p be any correct process active (and non-waiting) at a timeslot $\geq \tau^*$ and let τ_a denote the first timeslot $\geq \tau^*$ at which p is active and non-waiting.*

Then, $\mathsf{tr} \in \mathsf{log}(p, \tau_\mathrm{a})$.

If a protocol satisfies the ℓ-liveness property against a ρ-bounded static adversary, we say the protocol is *(ρ, ℓ)-live*.

Optimistic responsiveness. Informally, the optimistic responsiveness property guarantees that transactions are finalized at network speed whenever all processes are correct.

▶ **Definition 3** (ℓ_or-Responsiveness). *A (permissioned or quasi-permissionless) protocol satisfies* ℓ_or-responsiveness*, where $\ell_\mathrm{or} \in O(\delta)$, if and only if the following condition is satisfied for every timeslot $\tau \in \mathbb{N}_{\geq 1}$ in every execution where all processes are correct. Suppose the following holds:*

- *Let $\tau^* = \max(\tau, GST) + \ell_\mathrm{or}$.*
- *Let a transaction tr be received by a correct process at some timeslot $\leq \tau$.*
- *Let p be any correct process active (and non-waiting) at a timeslot $\geq \tau^*$ and let τ_a denote the first timeslot $\geq \tau^*$ at which p is active and non-waiting.*

Then, $\mathsf{tr} \in \mathsf{log}(p, \tau_\mathrm{a})$.

If a protocol satisfies the ℓ_or-optimistic responsiveness property against a ρ-bounded static adversary, we say that the protocol is *(ρ, ℓ_or)-responsive*. Let us briefly compare our definition of ℓ_or-responsiveness (Def. 3) and our definition of ℓ-liveness (Def. 2). As shown, $\ell_\mathrm{or} \in O(\delta)$, where δ denotes the actual (and unknown) upper bound on message delays after GST (cf. Sec. 2), while ℓ may depend on the known upper bound Δ and does not need to reflect actual network delays. Thus, while a responsive protocol can finalize transactions at the speed of the network, a live protocol may do so slower, depending on conservative bounds. Importantly, we note that our transformation ensures responsiveness even in non-failure-free executions assuming that all processes do behave correctly after GST.

Accountability. Accountability is a property ensuring that, if consistency is ever violated, a sufficient number of faulty processes are conclusively identified through undeniable proofs of guilt. We begin by introducing the concept of a proof of guilt in quasi-permissionless consensus algorithms; this definition is inspired by that from [65].

▶ **Definition 4** (Proof of guilt). *Let $\mathcal{P}$ be any quasi-permissionless protocol, and let $id \in \mathsf{IDs}$ be any identifier. Consider a set of messages $\mathcal{M}$, each of which is signed by id (i.e., using the corresponding private key). The set $\mathcal{M}$ constitutes a* proof of guilt *for id with respect to $\mathcal{P}$ if and only if there exists no execution of $\mathcal{P}$ in which (1) id (i.e., the corresponding process) sends all the messages from $\mathcal{M}$, and (2) id (i.e., the corresponding process) is correct.*

We are ready to define the accountability property in quasi-permissionless algorithms. Recall that our transformation guarantees accountability in the resulting quasi-permissionless protocol, regardless of whether the original permissioned protocol satisfies it.[4]

▶ **Definition 5** (ρ_{a}-Accountability). *A quasi-permissionless protocol satisfies* ρ_{a}-accountability *if and only if the following condition holds in every execution where there exist correct processes p and q, and timeslots $\tau_p \in \mathbb{N}_{\geq 1}$ and $\tau_q \in \mathbb{N}_{\geq 1}$ such that $\mathsf{log}(p, \tau_p)$ is inconsistent with $\mathsf{log}(q, \tau_q)$. Let $\mathcal{M}_p$ (resp., $\mathcal{M}_q$) be the set of all messages received by process p (resp., q) by timeslot τ_p (resp., τ_q). Let $\mathcal{F}$ be the set of identifiers for which a proof of guilt exists in $\mathcal{M}_p \cup \mathcal{M}_q$. Then, there must exist a correct process z and a timeslot $\tau_z \in \mathbb{N}_{\geq 0}$ such that the identifiers in $\mathcal{F}$ collectively hold at least a ρ_{a}-fraction of the total stake recorded in $\mathsf{log}(z, \tau_z)$:*

$$\sum_{id \in \mathcal{F}} \mathsf{S}(\mathsf{log}(z, \tau_z), id) \geq \rho_{\text{a}} \cdot \mathsf{log}(z, \tau_z).\mathsf{total\_stake} = \rho_{\text{a}} \cdot \mathbb{T}.$$

Let us analyze the definition of the ρ_{a}-accountability property. The property is "triggered" when a consistency violation occurs, meaning that two correct processes p and q output inconsistent logs. In such a case, the definition ensures that p and q collectively hold enough information to correctly identify as faulty a set of participants (i.e., identifiers) whose stake represents at least a ρ_{a}-fraction of the total stake.

Accountability vs. consistency. Accountability and consistency are inherently at odds. A ρ-consistent protocol guarantees that consistency is preserved as long as the adversary controls at most a ρ-fraction of the total stake; consistency may be violated only if this threshold is exceeded. Ideally, such a protocol would also achieve ρ_{a}-accountability for some $\rho_{\text{a}} > \rho$, meaning that whenever consistency is violated, the protocol can identify a set of faulty processes holding at least a ρ_{a}-fraction of the total stake. In other words, if consistency is violated, the adversary controls more than a ρ-fraction of the stake—hence, we seek to hold accountable a stake weight that exceeds the consistency threshold.

3.2 Composable Log-Specific Safety Properties

In this subsection, we define *composable log-specific safety properties*, a generic class of properties we translate from the permissioned to the quasi-permissionless setting.

Definition. A *log-specific property* P is a function $P : \mathbb{P}(\mathsf{Logs}) \to \{true, false\}$, where $\mathbb{P}$ denotes the power set and Logs denotes the set of all logs (cf. Sec. 2). Next, we define log-specific safety properties.

▶ **Definition 6** (Log-specific safety property). *A* log-specific safety property *S is a log-specific property with the following constraint:*

- *Let $logs \subseteq \mathsf{Logs}$ be any set of logs such that $S(logs) = false$. Then, there exists a finite subset $logs' \subseteq logs$ such that, for every set $logs''$ with $logs' \subseteq logs''$, $S(logs'') = false$.*

[4] That is why we define accountability solely with respect to quasi-permissionless protocols.

Intuitively, the constraint specifies that if a set of logs fails to satisfy S, there must exist a finite subset that also does not satisfy S, and none of its supersets satisfy S. We underline that Def. 6 follows the spirit of safety properties as defined by Alpern and Schneider in their seminal work [4]. We are now ready to define composable log-specific safety properties.

▶ **Definition 7** (Composable log-specific safety property)**.** *A log-specific safety property S is* composable *if and only if the following constraint holds:*

- *Let $(logs_1, logs_2) \subseteq \mathsf{Logs}^2$ be any pair of sets of logs such that $S(logs_1) = S(logs_2) = true$. Then, for every set $logs \subseteq logs_1 \cup logs_2$, $S(logs) = true$.*

In essence, a composable log-specific safety property S indicates that if two sets of logs satisfy S, then every subset of their union also satisfies S. Properties that require correct processes to output only valid logs (according to some predetermined validity condition) are composable log-specific safety properties. These include, for example, the following properties: (1) no log contains futile transactions, and (2) no log contains transactions not signed by their issuers.

Satisfying log-specific safety properties. Lastly, we define what it means for a protocol to satisfy a (composable or not) log-specific safety property. Fix any (permissioned or quasi-permissionless) protocol $\mathcal{P}$. Given any execution $\mathcal{E}$ of the protocol $\mathcal{P}$, let $\mathsf{logs}(\mathcal{E})$ denote the set of all logs held by correct processes in $\mathcal{E}$:

$$\mathsf{logs}(\mathcal{E}) \equiv \{\mathcal{L} \in \mathsf{Logs} \mid \exists p \in \Pi : p \text{ is correct} \wedge p \text{ outputs } \mathcal{L} \text{ in } \mathcal{E}\}.$$

Finally, we present the definition.

▶ **Definition 8** (Satisfying log-specific safety properties)**.** *Let S be any log-specific safety property and let $\mathcal{P}$ be any (permissioned or quasi-permissionless) protocol. We say that $\mathcal{P}$* satisfies *S if and only if, in every execution $\mathcal{E}$ with $S(\{\mathcal{L}_\mathrm{g}\}) = true$, where $\mathcal{L}_\mathrm{g}$ denotes the genesis log in $\mathcal{E}$, $S\big(\mathsf{logs}(\mathcal{E})\big) = true$.*

A protocol $\mathcal{P}$ satisfies a log-specific safety property S if correct processes output only logs that are allowed by S. Notably, we exclude executions in which the genesis log is not allowed by S; otherwise, no protocol could satisfy S as it is initially violated. We underline that time is irrelevant in the context of satisfying S: for any two (arbitrarily different) executions $\mathcal{E}_1$ and $\mathcal{E}_2$ that have the same set of output logs (i.e., $\mathsf{logs}(\mathcal{E}_1) = \mathsf{logs}(\mathcal{E}_2)$), the property S is either satisfied in *both* executions or in *neither*.

4 Transformation

In this section, we show how to transform any permissioned protocol $\mathcal{P}$ into a quasi-permissionless PoS protocol $\mathcal{T}(\mathcal{P})$. We begin by presenting an overview of our transformation (Sec. 4.1). Then, we introduce its pseudocode (Sec. 4.2). Finally, we provide an informal analysis of $\mathcal{T}(\mathcal{P})$'s correctness (Sec. 4.3). A formal proof is relegated to App. C.

4.1 Overview

We partition the execution of the quasi-permissionless PoS protocol $\mathcal{T}(\mathcal{P})$ into *epochs* (Fig. 1). In each epoch, the permissioned protocol $\mathcal{P}$ is executed, meaning that every epoch is responsible for producing a particular segment of the "global" log of finalized transactions. Each epoch runs for (at least) a predetermined duration greater than the liveness parameter of protocol $\mathcal{P}$. This ensures that, following GST, each epoch runs for sufficiently long to

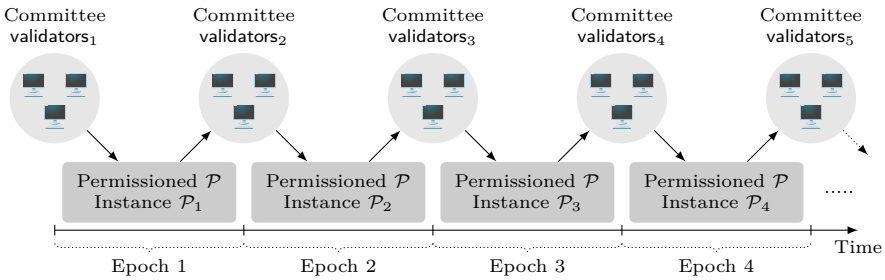

Figure 1 High-level overview of $\mathcal{T}(\mathcal{P})$'s epoch-based structure.

finalize new transactions. Once an epoch completes, the log produced up to (and including) that epoch is treated as the genesis log for the next epoch. Moreover, the identifiers holding stake according to the produced log validate the next epoch; the genesis log $\mathcal{L}_{\text{g}}$ determines the validators of the first epoch. During the entire execution, protocol $\mathcal{T}(\mathcal{P})$ keeps progressing through epochs, with each subsequent epoch building on the output of the previous one, resulting in an ever-growing log of finalized transactions.

Guarantees of $\mathcal{P}$ in $\mathcal{T}(\mathcal{P})$'s epoch-based structure. The correctness of our quasi-permissionless PoS protocol $\mathcal{T}(\mathcal{P})$ crucially relies on the guarantees provided by the permissioned protocol $\mathcal{P}$. However, we observe that, within the epoch-based structure of $\mathcal{T}(\mathcal{P})$, protocol $\mathcal{P}$ might display unexpected behavior as described below. In the standard permissioned setting, correct processes run $\mathcal{P}$ forever, as this is a fundamental characteristic of the setting. However, the epoch-based structure of $\mathcal{T}(\mathcal{P})$ introduces a different environment for $\mathcal{P}$: correct processes *stop* executing $\mathcal{P}$ associated with epoch e once they transition to epoch $e+1$. This minor change necessitates a re-evaluation of the guarantees offered by $\mathcal{P}$: $\mathcal{P}$'s guarantees in the standard permissioned setting do *not* directly extend to this new context.

To analyze $\mathcal{P}$'s behavior within the epoch-based structure of $\mathcal{T}(\mathcal{P})$, one needs to (1) enrich $\mathcal{P}$'s interface by allowing correct processes to stop executing it, and (2) examine the guarantees provided by this enriched protocol. We achieve this by introducing the concept of *quit-enhanced* permissioned protocols (cf. App. B.3): given any standard permissioned protocol $\mathcal{P}'$, where correct processes are expected to participate forever, we define $\mathit{quit}(\mathcal{P}')$ as the quit-enhanced version of $\mathcal{P}'$, where correct processes have the option to stop participating. (We emphasize that $\mathit{quit}(\mathcal{P})$ merely extends the interface of the original permissioned protocol $\mathcal{P}$. In particular, analyzing the behavior of $\mathit{quit}(\mathcal{P})$ relies exclusively on understanding the behavior of $\mathcal{P}$ itself—there is no need to examine the internal mechanisms of the protocol. This makes our quit-based extension inherently "closed-box" in nature.) Using our concept of quit-enhanced protocols, we prove that consistency, liveness, optimistic responsiveness, and all log-specific safety properties translate from $\mathcal{P}$ to $\mathit{quit}(\mathcal{P})$ (cf. App. B.4), allowing us to build $\mathcal{T}(\mathcal{P})$ on top of $\mathcal{P}$ that satisfies these properties in the standard permissioned model.[5]

[5] Liveness and optimistic responsiveness are preserved in only some (and not all) executions of $\mathit{quit}(\mathcal{P})$. However, as we show in App. C, only these executions are needed to prove $\mathcal{T}(\mathcal{P})$'s liveness and optimistic responsiveness.

Algorithm 1 Permissioned to PoS transformation $\mathcal{T}$: Pseudocode for process p [part 1/2]

```
1  Inputs:
2      Permissioned ρ-consistent protocol 𝒫 with liveness parameter ℓ        ▷ to be transformed
3  Constants:
4      Log 𝒞_g                                                              ▷ the genesis log; cf. Sec. 2
5      Integer 𝕋                                                            ▷ the total stake; cf. Sec. 2
6  Local variables:
7      Log log(p) ← 𝒞_g                                                     ▷ local log of p, initially set to genesis log
8      Set(Transactions) received_txs_p ← ∅                                 ▷ set of received transactions
9      Epoch epoch_p ← 0                                                    ▷ the current epoch
10     Set(IDs) current_validators_p ← ∅                                    ▷ set of p's identifiers validating the current epoch
11     Map(Log → Boolean) signed_p ← {false, for every log 𝒞 ∈ Logs}

12 at every timeslot τ:
13     received_txs_p ← the set of transactions received by timeslot τ (from the environment and
       other processes)
14     invoke feed(received_txs_p)                                          ▷ forward the transactions to simulated 𝒫
15     ▷ The next line can be optimized: p can only disseminate updates not previously disseminated.
16     ▷ For simplicity, we maintain the unoptimized pseudocode.
17     Disseminate fully-certified log(p) and received_txs_p

18 upon start:
19     invoke start_simulation(log(p))                                      ▷ start simulating the first epoch; log(p) = 𝒞_g here

20 upon receiving a fully-certified log 𝒞 such that 𝒞 extends log(p):      ▷ hence, 𝒞.epoch ≥ epoch_p
21     log(p) ← 𝒞                                                           ▷ update the local log
22     if 𝒞.epoch > epoch_p or 𝒞.completed = true then                       ▷ check if new epoch should be started
23         invoke stop_simulation                                           ▷ if so, stop simulation associated with the previous epoch
24         invoke start_simulation(log(p))                                  ▷ and start simulation associated with the new epoch

25 upon obtain_log(Log 𝒞):                                                  ▷ 𝒞 is obtained from simulated 𝒫
26     for each id ∈ current_validators_p:
27         for each Log 𝒞′ with 𝒞′.epoch = 𝒞.epoch and 𝒞 extends 𝒞′ and signed_p[𝒞′] = false:
28             if no log inconsistent with 𝒞′ has previously been signed then
29                 Sign 𝒞′ using the private key of id and disseminate 𝒞′ accompanied by the signature
30                 signed_p[𝒞′] ← true
```

4.2 Pseudocode

Transformation $\mathcal{T}$ takes a permissioned ρ-consistent protocol $\mathcal{P}$ satisfying liveness with parameter ℓ and a stake distribution function $\mathsf{S}(\cdot,\cdot)$ as input, and outputs a quasi-permissionless PoS protocol $\mathcal{T}(\mathcal{P})$ (cf. Fig. 2 and Alg. 1). Each epoch of $\mathcal{T}(\mathcal{P})$ is associated with a set of active identifiers, denoted by $\mathsf{validators}_e$. At the beginning of each epoch e, these identifiers execute an instance of $\mathcal{P}$ (ln. 24, ln. 31-ln. 90).

Executing $\mathcal{P}$ within an epoch e. As $\mathbb{T}$ denotes the total stake (cf. Sec. 2), identifiers utilized in (executed-in-an-epoch) permissioned protocol $\mathcal{P}$ are integers in the range $[1, \mathbb{T}]$; we refer to these identifiers as "permissioned-ids". Similarly, we refer to identifiers from the set IDs (cf. Sec. 2) as "PoS-ids".

Consider a correct process p executing $\mathcal{P}$ in epoch e. To start executing $\mathcal{P}$ in epoch e, process p invokes the $\mathsf{start\_simulation}(\cdot)$ function (ln. 55).[6] There, process p begins by setting the current log of the PoS protocol $\mathcal{T}(\mathcal{P})$, finalized at the end of the previous epoch $e-1$, as the genesis log $\mathcal{C}$ for epoch e (ln. 56-ln. 60). It then maps each permissioned-id (from the $[1, \mathbb{T}]$

[6] Throughout the remainder of this section and in the pseudocode, we say that processes simulate $\mathcal{P}$ to emphasize that this is the underlying permissioned protocol being executed, and that each PoS-id may be running $\mathcal{P}$'s state machine on behalf of multiple permissioned-ids.

Algorithm 1 Permissioned to PoS transformation $\mathcal{T}$: Pseudocode for process p [part 2/2]

```
▷ This section of the pseudocode is dedicated to simulating the permissioned protocol P.
Local variables:                                                  ▷ variables dedicated to simulating P
    Map([1, T] → IDs) id_map_p ← empty map                        ▷ permissioned-id to PoS-id
    Map([1, T] → Simulation) simulation_map_p ← empty map         ▷ permissioned-id to simulation
    Set(Simulation) simulations_p ← ∅                             ▷ set of p's currently executed simulations
    Timer epoch_timer_p                                           ▷ for measuring the duration of epochs

Local functions:
    Map([1, T] → IDs) map_stake(Logs L):                          ▷ returns permissioned-id to PoS-id mapping
        Map([1, T] → IDs) map ← empty map
        List(IDs) V ← L.current_ids                               ▷ find identifiers with positive stake according to L
        Sort V in lexicographical order
        Integer counter ← 1
        for each id ∈ V:                                          ▷ iterate through V in the ascending lexicographical order
            for each j ∈ [1, S(L, id)]:                           ▷ associate a number of permissioned-ids equal to the stake
                map[counter] ← id
                counter ← counter + 1
        return map

at every timeslot τ:
    if simulations_p ≠ ∅ then                                     ▷ check if p is validating the current epoch
        for each S ∈ simulations_p:
            Let L ← log(S.identifier, τ)                          ▷ obtain the current log of the simulated instance S
            if L.epoch > epoch_p then                             ▷ check if the current log is "too long"
                L ← L.ep_prefix(epoch_p)
            trigger obtain_log(L)                                 ▷ report the current log

upon start_simulation(Logs L):
    if L.completed = true then
        epoch_p ← L.epoch + 1                                     ▷ L is the genesis log for the epoch
    else                                                          ▷ L is a log from the "middle" of epoch
        epoch_p ← L.epoch                                         ▷ the current epoch is L's epoch
        L ← L.ep_prefix(epoch_p − 1)  ▷ (epoch_p − 1)-prefix of L is the genesis log for the epoch
    current_validators_p ← L.current_ids ∩ id(p)  ▷ set p's identifiers validating the current epoch
    id_map_p ← map_stake(L)                                       ▷ update the permissioned-id to PoS-id map
    for each i ∈ [1, T] such that id_map_p[i] ∈ id(p):
        Let simulation ← initialize P with permissioned-id i      ▷ instance of P with id i
        Start simulation with genesis log L   ▷ the genesis log for P with permissioned-id i is L
        simulations_p ← simulations_p ∪ {simulation}
        simulation_map_p[i] ← simulation
    if current_validators_p ≠ ∅ then
        invoke epoch_timer_p.measure(ED = ℓ + Δ)   ▷ if validating the epoch, start the timer

upon stop_simulation:
    for each S ∈ simulations_p:
        Stop executing S                                          ▷ stop each instance S of permissioned protocol P
    ▷ Reset the variables and cancel the timer
    invoke epoch_timer_p.cancel()
    id_map_p ← empty map
    simulation_map_p ← empty map
    simulations_p ← ∅
    current_validators_p ← ∅

upon epoch_timer_p expires:       ▷ the epoch should be completed, i.e., the timer has expired
    for each id ∈ current_validators_p:
        Disseminate the (FINISH, epoch_p) transaction issued by id

upon S ∈ simulations_p sends a message m:                         ▷ simulation instance S sends m
    M ← ⟨SIMULATION, epoch_p, m⟩                                  ▷ tag m with the current epoch
    ▷ Send the simulation message to the PoS-id associated with the permissioned-id m.receiver
    Send M to id_map_p[m.receiver]

upon receiving a simulation message M with M.epoch = epoch_p and simulations_p ≠ ∅:
    Forward M.message to simulation_map_p[M.message.receiver]

upon feed(Set(Transaction) txs):
    for each S ∈ simulations_p:
        Forward transactions txs to S                             ▷ simulation instance S receives transactions txs
```

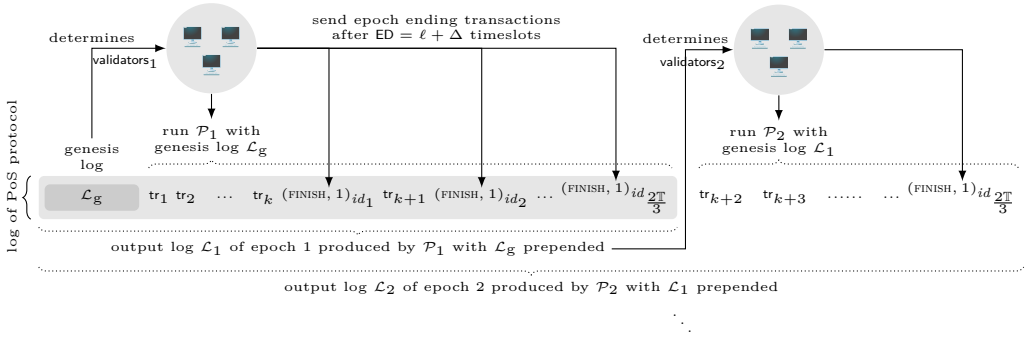

Figure 2 The quasi-permissionless PoS protocol $\mathcal{T}(\mathcal{P})$ proceeds in epochs (see Fig. 1). At each epoch e, a set of identifiers $\mathsf{validators}_e$ run a new instance $\mathcal{P}_e$ of the permissioned protocol $\mathcal{P}$; the genesis log for $\mathcal{P}_e$ is the log output in epoch $e-1$. $\mathsf{ED} = \ell + \Delta$ timeslots into the execution of $\mathcal{P}_e$, the validators issue a special epoch-ending transaction (FINISH, e). Once there are two-thirds-stake worth of finalized epoch-ending transactions, each validator stops executing the protocol $\mathcal{P}_e$. The log at that point determines the validators for the next epoch $e+1$, and is input as the genesis log to $\mathcal{P}_{e+1}$.

range) into one PoS-id (from the IDs set) using the $id\_map_p = \mathsf{map\_stake}(\mathcal{L})$ map (ln. 62). For instance, if $\mathcal{L}$ assigns 3 tokens to a PoS-id id, then there exist $x_1, x_2, x_3 \in [1, \mathbb{T}]$ such that $id\_map_p[x_1] = id\_map_p[x_2] = id\_map_p[x_3] = id$; intuitively, in epoch e, permissioned-ids x_1, x_2 and x_3 correspond to the PoS-id id, meaning that PoS-id id is responsible for simulating $\mathcal{P}$'s state machines associated with permissioned-ids x_1, x_2 and x_3. This is also crucial because each process must know which identifier to use when forwarding messages related to the permissioned protocol $\mathcal{P}$. Specifically, if a permissioned-id i_1 needs to communicate with another permissioned-id i_2 in $\mathcal{P}$ (associated with some epoch), the process p "responsible" for the permissioned-id i_1 must know a PoS-id corresponding to i_2 in order to forward the message correctly (ln. 87). After establishing the aforementioned permissioned-ids to PoS-ids mapping, process p verifies whether it is validating epoch e, i.e., whether any of its identifiers have been assigned a positive stake by $\mathcal{L}$. If there exists a permissioned-id $x \in [1, \mathbb{T}]$ that maps into a p's PoS-id $id \in \mathsf{id}(p)$, process p instantiates $\mathcal{P}$'s state machine initialized with permissioned-id x and genesis log $\mathcal{L}$ (lns. 64 and 65). Then, p updates $simulation\_map_p[x]$ to the initialized state machine (ln. 67). Note that id_map_p associates each permissioned-id with its PoS-id counterpart, whereas $simulation\_map_p$ maps only p's permissioned-ids into their respective state machines. Finally, if p is indeed validating epoch e, it instructs its timer $epoch\_timer_p$ to measure $\mathsf{ED} = \ell + \Delta$ time (ln. 69). Here, $\mathsf{ED} = \ell + \Delta$ is selected large enough so that each simulated instance $\mathcal{P}$ is run sufficiently long to allow new transactions to be finalized after GST. Specifically, the time period $\mathsf{ED} = \ell + \Delta$ ensures that, after GST, all correct validators of some post-GST epoch e overlap in their execution of $\mathcal{P}$ (associated with epoch e) for at least ℓ time. This overlap, together with the ℓ-liveness property of $\mathcal{P}$, guarantees that new transactions are finalized.

Outputting logs. Within every epoch e, processes output a log produced by $\mathcal{P}$ as the $\mathcal{T}(\mathcal{P})$ log if the log is signed by any quorum of the set $\mathsf{validators}_e$ (ln. 20 and Def. 25).

Epoch change. When $epoch\_timer_i$ expires, process p disseminates a special *epoch-ending* transaction (FINISH, e) on behalf of each of its validating PoS-ids (ln. 81). The purpose of these FINISH transactions is ensuring that epoch e eventually concludes by producing a complete log (cf. Def. 23), i.e., an epoch-ending block.

Process p stops simulating $\mathcal{P}$ upon observing epoch-ending transactions from a quorum

of identifiers with sufficient stake, i.e., some quorum $I \subseteq \{id | id \in \mathsf{validators}_e\}$ such that $\sum_{id \in I} \mathsf{S}(\mathcal{L}, id) \geq (1-\rho)\mathbb{T}$ (ln. 24 and Def. 23). At that point, p stops each of its currently simulated state machines (ln. 72), cancels $epoch\_timer_p$ (ln. 74), and resets the simulation-specific variables (ln. 75-ln. 78).

Determining the next set of validators. The set $\mathsf{validators}_{e+1}$ of validators is selected based on the stake distribution determined by the log produced in epoch e (ln. 62 and Def. 21). More specifically, any identifier that has positive stake according to the log $\mathcal{L}$ produced in epoch e is eligible to participate in the permissioned protocol instance $\mathcal{P}$ associated with the next epoch $e+1$.

Methods defined on logs. The pseudocode of our transformation $\mathcal{T}$ utilizes many methods defined on logs. Their formal definition can be found in App. C.1, while an informal description is given below:

- $\mathcal{L}$.current_ids: the set of identifiers with positive stake according to $\mathcal{L}$.
- $\mathcal{L}$.epoch: the epoch with which $\mathcal{L}$ is associated.
- $\mathcal{L}$.validators: the set of identifiers validating the epoch of $\mathcal{L}$, i.e., the set of identifiers that "produced" $\mathcal{L}$.
- $\mathcal{L}$.completed: *true* if and only if $\mathcal{L}$ contains sufficiently many (i.e. $(1-\rho)\mathbb{T}$ worth of stake) epoch-ending transactions.
- $\mathcal{L}$.ep_prefix$\big(e \in [0, \mathcal{L}.\mathsf{epoch})\big)$: the completed log $\mathcal{L}'$ of epoch e such that $\mathcal{L}$ extends $\mathcal{L}'$.

Moreover, we say that a log $\mathcal{L}$ is *certified* at a process p if and only if p receives signatures on the log $\mathcal{L}$ from a quorum of members (i.e. $(1-\rho)\mathbb{T}$ worth of stake) of $\mathcal{L}$.validators. Finally, a certified log $\mathcal{L}$ is *fully-certified* at a process p if and only if, for every $e \in [0, \mathcal{L}.\mathsf{epoch})$, $\mathcal{L}$.ep_prefix(e) is certified at process p.

4.3 Correctness

In this subsection, we provide an informal analysis of the correctness of our transformation.

Consistency. The following theorem proves that our transformation preserves the consistency property from the permissioned to the quasi-permissionless setting.

▶ **Theorem 9** (Consistency). *Consider a permissioned protocol $\mathcal{P}$ that satisfies consistency against a ρ-bounded adversary. Then, the quasi-permissionless protocol $\mathcal{T}(\mathcal{P})$ satisfies consistency against a ρ-bounded adversary.*

Proof sketch. We show that the consistency property translates from $\mathcal{P}$ to $\mathcal{T}(\mathcal{P})$ by induction. Initially, all correct processes have $\mathcal{L}_{\mathrm{g}}$ as the genesis log. Given that the logs output by the first epoch are consistent (due to $\mathcal{P}$'s consistency), consistency is satisfied in the first epoch. Moreover, all correct processes agree on the genesis log and the set of validators for the second epoch. By inductively applying the same argument, we can show that the logs output by the correct processes remain consistent throughout the entire execution. A formal proof of the theorem is given in App. C.3. □

Composable log-specific safety properties. The theorem below asserts that our transformation carries over any composable log-specific safety property from the permissioned setting to the quasi-permissionless setting.

▶ **Theorem 10** (Composable log-specific safety property). *Consider a permissioned protocol $\mathcal{P}$ that satisfies consistency and some composable log-specific safety property S against a ρ-bounded adversary. Then, $\mathcal{T}(\mathcal{P})$ satisfies S against a ρ-bounded adversary.*

Proof sketch. To prove that any composable log-specific safety property S translates from $\mathcal{P}$ to $\mathcal{T}(\mathcal{P})$, we again rely on induction. As $\mathcal{P}$ satisfies S, the set of logs output by the first epoch adheres to S. Similarly, the set of logs output by the second epoch adheres to S. Hence, their union adheres to S as S is composable (cf. Def. 7):

$$S(\{\mathcal{L} \mid \mathcal{L}.\mathsf{epoch} \in \{1,2\} \wedge \mathcal{L} \text{ is output by a correct process}\}) = \mathit{true}.$$

Again, the set of logs output by the third epoch adheres to S, which then implies:

$$S(\{\mathcal{L} \mid \mathcal{L}.\mathsf{epoch} \in \{1,2,3\} \wedge \mathcal{L} \text{ is output by a correct process}\}) = \mathit{true}.$$

By inductively applying this argument, we can show that $\mathcal{T}(\mathcal{P})$ satisfies S. A formal proof of the theorem is given in App. C.4. □

Liveness & optimistic responsiveness. The following theorem demonstrates the preservation of liveness and optimistic responsiveness by our transformation.

▶ **Theorem 11** (Liveness & optimistic responsiveness)**.** *Consider a permissioned protocol $\mathcal{P}$ that satisfies consistency and ℓ-liveness, for some $\ell < \infty$, against a ρ-bounded adversary. Then, $\mathcal{T}(\mathcal{P})$ satisfies $\ell^\star$-liveness, with $\ell^\star = 2\Delta + 2\ell$, against a ρ-bounded adversary. Furthermore, if $\mathcal{P}$ additionally satisfies ℓ_{or}-responsiveness, for some $\ell_{\text{or}} \in O(\delta)$, where δ denotes the actual bound on message delays after GST, against a ρ-bounded adversary, then $\mathcal{T}(\mathcal{P})$ satisfies $\ell^\star_{\text{or}}$-responsiveness, with $\ell^\star_{\text{or}} = 2\delta + 2\ell_{\text{or}}$, against a ρ-bounded adversary.*

Proof sketch. If the first correct process p for which there exists an identifier $id \in \mathsf{id}(p) \cap \mathsf{validators}_e$ enters an epoch e at some timeslot $\tau \geq \text{GST}$, then all other correct validators enter epoch e within Δ timeslots after observing the log seen by p. Moreover, no correct process sends an epoch-ending transaction until $\mathsf{ED} = \ell + \Delta$ timeslots into epoch e, which implies that epoch e cannot be completed before timeslot $\tau + \mathsf{ED} = \tau + \ell + \Delta$. Hence, all correct validators stay in epoch e together for at least ℓ timeslots. As $\mathcal{P}$ satisfies ℓ-liveness, this overlap is sufficient to finalize new transactions in epoch e. To prove optimistic responsiveness, we follow the previous argument while factoring in that the actual message delay bound is δ. As a result, all correct validators join epoch e within δ timeslots—rather than $\Delta > \delta$—after seeing the log observed by p. A formal proof is relegated to App. C.5. □

Accountability. Finally, the following theorem demonstrates that our transformation ensures that the resulting quasi-permissionless protocol satisfies accountability. Note that this holds even if the original permissioned protocol lacks accountability.

▶ **Theorem 12** (Accountability)**.** *Consider a permissioned protocol $\mathcal{P}$ that satisfies consistency against a ρ-bounded adversary. Then, the quasi-permissionless protocol $\mathcal{T}(\mathcal{P})$ satisfies $(1-2\rho)$-accountability.*

Proof sketch. Suppose two correct processes p and q output inconsistent logs $\mathcal{L}_p$ and $\mathcal{L}_q$, respectively. Therefore, $\mathcal{L}_p$ (resp., $\mathcal{L}_q$) is fully-certified at p (resp., q). Hence, disseminating these two logs, along with their respective signatures, enables accountability: every correct process eventually receives these inconsistent logs, combines the received signatures, and obtains a set of identifiers C such that, for each $id \in C$, id signs two inconsistent logs associated with the same epoch, thus proving id's culpability. Finally, since each of the two logs is signed by a quorum of validators—i.e., those holding at least $(1-\rho)\mathbb{T}$ stake—the identifiers in C collectively represent at least $(1-\rho)\mathbb{T} + (1-\rho)\mathbb{T} - \mathbb{T} = (1-2\rho)\mathbb{T}$ stake. A formal proof of the theorem can be found in App. C.6. □

In terms of message complexity, the transformation $\mathcal{T}(\mathcal{P})$ introduces an additional quadratic term to that of $\mathcal{P}$ in order to satisfy the accountability property in the quasi-permissionless setting, stemming from the signatures that processes must exchange before outputting a log. However, since any consensus protocol inherently requires a quadratic number of messages in the worst case [37], our transformation does not introduce any asymptotic worst-case overhead. A proof is deferred to App. C.7.

5 Extensions

We now present some extensions that can further improve our transformation.

Beyond public key infrastructure. In this work, we assumed the use of digital signatures (cf. Sec. 2). However, many permissioned protocols rely on "heavier" cryptographic primitives (e.g., [86, 63, 28]) such as threshold signatures. Importantly, our transformation can easily be adapted for these protocols, provided that the cryptographic primitives necessary for the underlying permissioned protocol are established in each epoch. Concretely, any setup procedure necessary for the permissioned protocol being transformed should be treated as an integral *part* of the permissioned protocol itself. Finally, we note that, to defeat long range attacks [36], key-erasure techniques [26] can be employed on the PoS protocol, so that the processes that have become passive, even if corrupted in the future, cannot create a log for the past epochs in retrospect.

Establishing the EAAC property. Our transformation can be extended to enable the EAAC ("expensive to attack in the absence of collapse") property introduced in [64]. Informally, the EAAC property captures a strong form of security in PoS protocols: it guarantees that launching a successful attack is prohibitively costly. More precisely, if an adversary attempts to violate the protocol's guarantees, the property ensures that the faulty participants responsible for the attack will be penalized by having their stake slashed. At the same time, the property protects correct participants by ensuring they remain unharmed and retain their stake, even during adversarial conditions. Given the technical complexity and detailed formalism of the EAAC property, we omit the full formal definition here and instead provide the aforementioned informal description. We encourage interested readers to consult [64] for the complete and rigorous treatment of the EAAC property.

Obtaining EAAC using our transformation. It is established in [64] that no non-trivial EAAC guarantees can be provided by any quasi-permissionless protocol within the standard partially synchronous model: if the adversary is strong enough to cause consistency violations, it can also evade punishment. However, the same work demonstrates that, under a stronger notion of partial synchrony—one that imposes a pessimistic upper bound on message delays Δ' (potentially orders of magnitude greater than Δ) even before GST—it becomes possible to design a quasi-permissionless protocol that satisfies the EAAC property, assuming the adversary controls less than two-thirds of the total stake in every execution. We follow this approach: our transformation, in addition to the properties already discussed, enables the resulting quasi-permissioned protocol to satisfy the EAAC property assuming that this pessimistic bound Δ' on message delays (even before GST) holds. Importantly, the resulting protocol satisfies the EAAC property regardless of whether the original permissioned protocol satisfies it or not.

Let us now provide modifications to our transformation $\mathcal{T}(\mathcal{P})$ sufficient for satisfying the EAAC property (assuming the pessimistic bound Δ' on message delays):

1. Each epoch e is executed for a duration (at least) proportional to the pessimistic bound on message delays Δ'. Concretely, each epoch is executed for more than $2\Delta'$ time.

2. When a process p validating epoch e receives a fully-certified log $\mathcal{L}$, the process (1) signs a message $m = \langle \text{CONFIRM}, \mathcal{L} \rangle$, (2) sends m to all identifiers validating epoch e, and (3) disseminates the received signatures (that fully-certified $\mathcal{L}$). We underline that modified $\mathcal{T}(\mathcal{P})$ includes two rounds of voting on a log: the first makes logs fully-certified (and is present in original, EAAC-less $\mathcal{T}(\mathcal{P})$), and the second revolves around the aforementioned CONFIRM messages (unique to $\mathcal{T}(\mathcal{P})$ modified for EAAC).
3. When a process p receives two-thirds-stake worth of CONFIRM messages for some log $\mathcal{L}$, process p "packs together" the received CONFIRM signatures and disseminates $\mathcal{L}$ along with the signatures.
4. A process p outputs a log $\mathcal{L}$ (i.e., sets $\mathsf{log}(p)$ to $\mathcal{L}$) only upon receiving two-thirds-stake worth of signatures on $\langle \text{CONFIRM}, \mathcal{L} \rangle$.
5. Finally, once a consistency violation occurs, the processes repeatedly initiate instances of the Dolev-Strong protocol [38]. This protocol is used to collectively agree on an updated genesis block, which incorporates slashing penalties for any processes identified as faulty during the violation. When this updated genesis block is established, the system can safely resume normal execution from this new agreed-upon state. This recovery procedure follows the approach detailed in [64, Algorithm 2].

Why do these modifications enable the EAAC property? If there is a consistency violation in an epoch e, there exists a correct process p (resp., q) validating epoch e that receives two-thirds-stake worth of signatures on some log $\mathcal{L}_p$ (resp., $\mathcal{L}_q$) *while being in epoch* e; these are the signatures making inconsistent logs $\mathcal{L}_p$ and $\mathcal{L}_q$ fully-certified. As (1) both p and q disseminate the received signatures, (2) message delays are bounded by Δ' even before GST, and (3) epoch $e + 1$ is executed for a period of time proportional to Δ', all validators of epoch $e + 1$ receive the conflicting signatures while in epoch $e + 1$ and ensure the slashing of the responsible identifiers (via the Dolev-Strong recovery procedure). Moreover, no correct process is ever slashed as no correct process (i.e., validator) ever signs conflicting logs, which means that no proof of guilt of a correct process could ever be produced.

6 Related Work

Due to space constraints, expansive comparison to related work, including on *group membership & view synchronous communication* [14, 22, 27, 14, 76, 6, 7, 67, 16] and on *deterministic reconfiguration in asynchrony* [43, 2, 3, 50, 56, 44, 81, 47, 23, 57], is relegated to App. D.

For *PoS blockchain protocols*, Ethereum [42, 20, 21] is the largest PoS blockchain by market cap, but the idea of PoS traces back to the Bitcoin community [11, 12, 83, 85, 84, 66, 54, 68]. The first provably-secure PoS protocols include SnowWhite [33, 74] and Ouroboros Praos [35, 53]. Hybrid Consensus [73, 55] proceeds in epochs, each of which has an associated permissioned consensus instance, with epoch transition and output log construction similar to that of our transformation. Lewis–Pye and Roughgarden [64] and Budish et al. [19] transform HotStuff [86] and Tendermint [18], respectively, from the permissioned to the PoS setting. PaLa [25] is a partially synchronous protocol with built-in reconfiguration. Sui Lutris [15] features a unique reconfiguration mechanism for its combined partially-ordering/totally-ordering consensus mechanism [78, 48, 10]. There is a substantial line of work on *reconfiguration of replicated state-machines* [59, 61, 62, 60, 1]. Systems implementing reconfiguration include BFT-SMaRt [13] and Raft [72]. Duan and Zhang [40] propose Dyno, a family of total-order broadcast protocols with reconfiguration carefully woven in in a bespoke manner. To the best of our knowledge, no earlier work describes a closed-box transformation from permissioned to PoS consensus that preserves and provides the range of desirable

properties studied in this paper.

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A System Model: Detailed Description

This section describes our model in full detail. (Recall that the overview is given in Sec. 2.) We start by introducing processes and how they communicate (App. A.1). Then, we define the standard partially synchronous model of communication, the environment that issues transactions to processes and the adversary (App. A.2). Finally, we define permissioned and quasi-permissionless protocols with a particular focus on the knowledge processes possess within these protocols (App. A.3).

A.1 Processes & Means of Communication

Processes. We consider a (potentially infinite) set of processes denoted by Π. Each process $p \in \Pi$ is assigned a non-empty and potentially infinite set of *identifiers*, denoted by $\mathsf{id}(p)$. Intuitively, $\mathsf{id}(p)$ determines the set of public keys for which process p knows the corresponding private key (cf. the "Public key encryption scheme" paragraph below); process p can use its identifiers to create sybils. Importantly, identifier sets are disjoint:

$$\forall (p_1, p_2) \in \Pi^2 : p_1 \neq p_2 \implies \mathsf{id}(p_1) \cap \mathsf{id}(p_2) = \emptyset.$$

We denote by IDs the set of all identifiers. We note that there may exist an identifier $id \in \mathsf{IDs}$ not allocated to any process, i.e., $\forall p \in \Pi : id \notin \mathsf{id}(p)$.

Time & process allocation. Time is divided into discrete timeslots whose range is $\mathbb{N}_{\geq 0}$. Each process may or may not be *active* at each timeslot. To accommodate for clock drifts, our model permits processes to be idle even at timeslots at which they are active. Concretely, at each timeslot at which a process is active, the process can either be *waiting* or *not waiting*. (The slower a process's clock, the more frequently the process waits at active timeslots.) A *process allocation* is a function specifying, for each process $p \in \Pi$, the timeslots at which process p is active and, for each such timeslot, if the process is waiting or not.

Inputs. Each process $p \in \Pi$ is given a finite set of *inputs*, which captures its knowledge at the beginning of the execution. If a variable is specified as one of p's inputs, we refer to it as *determined for* p; otherwise, the variable is *undetermined for* p. If a variable is determined (resp., undetermined) for every process $p \in \Pi$, we say that the variable is *determined* (resp., *undetermined*).

Public key infrastructure. In this work, we focus on protocols assuming a public key infrastructure (PKI) that allows processes to sign their messages and verify messages received from other processes. Concretely, each process p associates each $id \in \mathsf{id}(p)$ with a pair of *private* and *public keys*; the public key is disseminated to other processes, whereas the private key is kept secret. Note that processes associate public keys with identifiers, and not with processes as the identifier function $\mathsf{id}(\cdot)$ is unknown (cf. App. A.3 for more details). A process p can send a message m signed using the private key of an identifier id if and only if (1) $id \in \mathsf{id}(p)$, or (2) p has previously received m and an id's signature on m.

Communication. At each timeslot, each non-waiting active process may *send* a finite set of messages on behalf of (any) one of its identifiers to any other specific identifier. Intuitively, we assume that each identifier is associated with its IP address, enabling messages to be sent to that identifier directly. Similarly, at each timeslot, each non-waiting active process may

receive (a possibly empty) set of messages sent to its identifiers. Waiting active or inactive processes do not send or receive any messages.

We also assume that processes may send their messages to the entire universe of processes. Specifically, at each timeslot, each non-waiting active process may *disseminate* a finite set of messages on behalf of (any) one of its identifiers to *all* other identifiers. In practice, this dissemination can be implemented by some version of a gossip protocol [52].

A.2 Partial Synchrony & Adversary

Partial synchrony. In this work, we consider the standard partially synchronous model [41]. Informally, the partially synchronous model is a hybrid between full asynchrony and full synchrony: an unknown timeslot exists where the system switches from acting asynchronously to behaving synchronously. Formally, each execution is associated with an unknown timeslot $\text{GST} \in \mathbb{N}_{\geq 0}$ such that the following two conditions hold:

- If any correct process p is active at any timeslot $\tau \geq \text{GST}$, then p is not waiting at timeslot τ. Intuitively, our model forbids local clocks from drifting after GST, while allowing them to drift before GST. (We emphasize that all our results can easily be extended to the model in which bounded clock drifts occur even after GST.)
- There exists known duration $\Delta \in \mathbb{N}_{\geq 1}$ such that the following is satisfied:
  - If (1) a process sends a message at any timeslot τ to any identifier id, and (2) process p with $id \in \mathsf{id}(p)$ is active and non-waiting at some timeslot $\tau' \geq \max(\tau, \text{GST}) + \Delta$, then p receives the message at a timeslot $\leq \tau'$.
  - If (1) a process disseminates a message at any timeslot τ, and (2) another process p is active and non-waiting at some timeslot $\tau' \geq \max(\tau, \text{GST}) + \Delta$, then p receives that dissemination at a timeslot $\leq \tau'$.

  That is, message delays are bounded by Δ after GST.

Lastly, each execution is associated with an unknown duration δ that denotes the *actual* bound on message delays. Concretely, $\delta \in \mathbb{N}_{\geq 1}$ is the smallest duration such that if (1) a process sends a message at any timeslot τ to any identifier id, and (2) process p with $id \in \mathsf{id}(p)$ is active and non-waiting at some timeslot $\tau' \geq \max(\tau, \text{GST}) + \delta$, then p receives the message at a timeslot $\leq \tau'$. The same holds for disseminated messages. Naturally, $\delta \leq \Delta$.

Adversary. We consider a static adversary that corrupts a fraction of all processes at the beginning of each execution. A corrupted process is said to be *faulty*, whereas a non-faulty process is said to be *correct*. Faulty processes may behave arbitrarily (i.e., we consider Byzantine failures), while correct processes adhere strictly to the prescribed protocol.

A.3 Permissioned Setting vs. Quasi-Permissionless Setting

We now examine the additional constraints that arise in each of the two settings.

Permissioned setting. In the permissioned setting, the set of processes Π is finite and known. Concretely, $|\Pi| = n$, for some $n \in \mathbb{N}_{\geq 1}$; the number of processes n is also known. Moreover, each process has a single identifier:

$$\forall p \in \Pi : \mathsf{id}(p) = \{p\}.$$

Moreover, the identifier function $\mathsf{id}(p)$ is also known. Lastly, each process is active at every timeslot.

Defining static ρ-bounded adversaries. A static ρ-bounded adversary, for any $\rho \in [0, 1]$, corrupts at most $\rho \cdot n$ processes at the beginning of each execution.

Determined vs. undetermined inputs. The following inputs are determined:
- the set of processes Π, its finite cardinality $n = |\Pi|$, and the identifier function $\mathsf{id}(\cdot)$;
- the bound on the power of the adversary ρ, the stake distribution function $\mathsf{S}(\cdot, \cdot)$, and the genesis log;
- the upper-bound on message delays Δ.

In contrast, the following inputs are undetermined:
- the set of corrupted processes and its cardinality;
- GST and the actual bound on message delays $\delta \leq \Delta$;
- the process allocation function: any specific process does not know if any other specific process is waiting or not at any specific timeslot τ (as the process does not know if $\tau \geq \text{GST}$).

Quasi-permissionless setting. In the quasi-permissionless setting, the set of process Π is not necessarily finite. Moreover, processes might have more than a single associated identifier. In the quasi-permissionless setting, only processes with non-zero stake are guaranteed to be active. Specifically, for any timeslot τ and any correct process p for which there exist a τ-active correct process q and an identifier $id_p \in \mathsf{id}(p)$ with $\mathsf{S}(\mathsf{log}(q, \tau), id_p) > 0$, process p is active at τ.

Defining static ρ-bounded adversaries. For every correct process p and every timeslot τ, for any $\mathcal{L}$ that is a prefix of $\mathsf{log}(p, \tau)$, at most ρ fraction of $\mathcal{L}$'s total stake (i.e., $\mathcal{L}.\mathsf{total\_stake} = \mathbb{T}$) belongs to identifiers associated with faulty processes according to the stake distribution specified by $\mathcal{L}$.

Determined vs. undetermined inputs. In the quasi-permissionless setting, the following inputs are determined:
- the bound on the power of the adversary ρ, the stake distribution function $\mathsf{S}(\cdot, \cdot)$, and the genesis log;
- the upper-bound on message delays Δ.

The following inputs are undetermined:
- the set of processes Π, its cardinality, and the identifier function $\mathsf{id}(\cdot)$;
- the set of corrupted processes and its cardinality;
- GST and the actual bound on message delays $\delta \leq \Delta$;
- the process allocation function.

B Permissioned Blockchain Protocols

In this section, we present a formal analysis of permissioned blockchain protocols. We begin by defining the computational model underlying permissioned blockchain protocols (App. B.1). Next, we formally model executions of these protocols (App. B.2). We then introduce the definitions of both standard and quit-enhanced permissioned blockchain protocols (App. B.3). Finally, we prove that the quit-enhanced protocols maintain the key properties of their standard counterparts (App. B.4). For simplicity, any reference to a "blockchain protocol" in this section implies a permissioned blockchain protocol.

B.1 Computational Model

Formally, a blockchain protocol $\mathcal{P}$ is a tuple

$$(\Pi, \rho, \mathsf{States}, \{s_p^0 \mid p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}, \mathsf{Logs}, \mathsf{Transition}),$$

as we explain below.

Processes & resilience. We denote by Π the set of processes executing the blockchain protocol $\mathcal{P}$. Moreover, we denote by $\rho \in [0, 1]$ the resilience of $\mathcal{P}$; we say that $\mathcal{P}$ is a ρ-resilient blockchain protocol.

States. We denote by States the set of states processes can take while executing $\mathcal{P}$. Each state $s \in \mathsf{States}$ encodes the following information:

- the process associated with s, denoted by $s.\mathsf{process}$;
- the log associated with s (might be $\bot$), denoted by $s.\mathsf{log}$;
- if s is the special quitting state, denoted by $s.\mathsf{quit} \in \{true, false\}$.

For each process $p \in \Pi$, there exists the *initial state* $s_p^0 \in \mathsf{States}$ such that (1) $s_p^0.\mathsf{process} = p$, (2) $s_p^0.\mathsf{log} = \bot$, and (3) $s_p^0.\mathsf{quit} = false$. For every non-initial state $s \in \mathsf{States}$, $s.\mathsf{log} \neq \bot$.

Messages. We denote by $\mathsf{Messages}$ the set of messages processes can send and receive while executing $\mathcal{P}$. Each message $m \in \mathsf{Messages}$ encodes the following information:

- the sender of m, denoted by $m.\mathsf{sender}$;
- the receiver of m, denoted by $m.\mathsf{receiver}$.

Transactions. We denote by $\mathsf{Transactions}$ the set of transactions processes can receive from the environment while executing $\mathcal{P}$. Each transaction $tx \in \mathsf{Transactions}$ encodes the following information:

- the process that receives tx, denoted by $tx.\mathsf{process}$;
- if tx is the special starting transaction, denoted by $tx.\mathsf{start} \in \{true, false\}$;
- if tx is the special quitting transaction, denoted by $tx.\mathsf{quit} \in \{true, false\}$;
- the log associated with tx (might be $\bot$), denoted by $tx.\mathsf{log}$.

The following holds for each transaction $tx \in \mathsf{Transactions}$: $tx.\mathsf{log} \neq \bot$ if and only if $tx.\mathsf{start} = true$. Moreover, there is no transaction $tx \in \mathsf{Transactions}$ such that $tx.\mathsf{start} = tx.\mathsf{quit} = true$. Finally, for each process $p \in \Pi$ and each log $\mathcal{L} \in \mathsf{Logs}$, there exists a transaction $tx \in \mathsf{Transactions}$ such that (1) $tx.\mathsf{process} = p$, (2) $tx.\mathsf{start} = true$, (3) $tx.\mathsf{quit} = false$, and (4) $tx.\mathsf{log} = \mathcal{L}$.

State-transition function. We denote by $\mathsf{Transition}$ the state-transition function of $\mathcal{P}$ that maps (1) a state, (2) a boolean, (3) a set of messages, and (4) a set of transactions into (a) a new state, and (b) a set of messages. Formally, given (1) a state $s \in \mathsf{States}$, (2) a set of messages $M^R \subseteq \mathsf{Messages}$ such that, for every message $m \in M^R$, $m.\mathsf{receiver} = s.\mathsf{process}$, and (3) a set of transactions $T^R \subseteq \mathsf{Transactions}$ such that for every transaction $tx \in T^R$, $tx.\mathsf{process} = s.\mathsf{process}$, $\mathsf{Transition}(s, waiting, M^R, T^R) = (s', M^S)$, for some s' and M^S, where

- $s' \in \mathsf{States}$ is a state for which $s'.\mathsf{process} = s.\mathsf{process}$;
- $M^S \subsetneq \mathsf{Messages}$ is a set of messages such that, for every message $m \in M^S$, $m.\mathsf{sender} = s.\mathsf{process}$;
- if $waiting = true$, then $s' = s$ and $M^S = \emptyset$.

Given $\mathsf{Transition}(s, waiting, M^R, T^R) = (s', M^S)$, the following holds:

- if $waiting = true$, then $s' = s$ and $M^S = \emptyset$;
- if $s.\mathsf{quit} = false$ and there does not exist a transaction $tx \in T^R$ with $tx.\mathsf{quit} = true$, then $s'.\mathsf{quit} = false$;
- if $s.\mathsf{quit} = false$ and there exists a transaction $tx \in T^R$ with $tx.\mathsf{quit} = true$, then $s'.\mathsf{quit} = true$, $s'.\mathsf{log} = s.\mathsf{log}$ and $M^S = \emptyset$.
- if $s.\mathsf{quit} = true$, then $s' = s$ and $M^S = \emptyset$;
- if $s = s_p^0$, for some process $p \in \Pi$, and there exists a transaction $tx \in T^R$ with $tx.\mathsf{start} = true$, then $s'.\mathsf{log} = tx.\mathsf{log}$;
- if $s = s_p^0$, for some process $p \in \Pi$, and there does not exist a transaction $tx \in T^R$ with $tx.\mathsf{start} = true$, then $s' = s$ and $M^S = \emptyset$.

B.2 Behaviors & Executions

We now model executions of blockchain protocols. Hence, fix any such protocol $\mathcal{P} = (\Pi, \rho, \mathsf{States}, \{s_p^0 \,|\, p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}, \mathsf{Logs}, \mathsf{Transition})$.

Fragments. A tuple $\mathcal{FR} = \left(s, \tau, \mathit{waiting}, M^R, T^R, M^S\right)$, where

- $s \in \mathsf{States}$ is a state;
- $\tau \in \mathbb{N}_{\geq 0} \cup \{+\infty\}$ is a timeslot;
- $\mathit{waiting} \in \{\mathit{true}, \mathit{false}\}$;
- $M^R \subsetneq \mathsf{Messages}$ is a set of messages;
- $T^R \subsetneq \mathsf{Transactions}$ is a set of transactions;
- $M^S \subsetneq \mathsf{Messages}$ is a set of messages,

is a *τ-fragment* of a process $p \in \Pi$ according to the protocol $\mathcal{P}$ if and only if:

1. $s.\mathsf{process} = p$;
2. for every message $m \in M^R$, $m.\mathsf{receiver} = p$;
3. for every transaction $tx \in T^R$, $tx.\mathsf{process} = p$;
4. for every message $m \in M^S$, $m.\mathsf{sender} = p$;
5. if $\mathit{waiting} = \mathit{true}$, then $M^R \cup T^R \cup M^S = \emptyset$.

Additionally, we say that $\mathcal{FR}$ is a *valid* τ-fragment of process p according to the protocol $\mathcal{P}$ if and only if:

- if $\tau = 0$, then $s = s_p^0$;
- no two transactions $tx_1, tx_2 \in T^R$ exist such that $tx_1.\mathsf{start} = \mathit{true}$ and $tx_2.\mathsf{quit} = \mathit{true}$;
- if $s = s_p^0$, then $\mathit{waiting} = \mathit{false}$;
- if $s = s_p^0$, then $M^R \cup T^R = \emptyset$ or $M^R \cup T^R = \{tx\}$, for some transaction $tx \in \mathsf{Transactions}$ with $tx.\mathsf{start} = \mathit{true}$;
- if $s = s_p^0$ and there does not exist a transaction $tx \in T^R$, then $M^S = \emptyset$;
- $\mathsf{Transition}(s, \mathit{waiting}, M^R, T^R) = (\cdot, M^S)$.

Intuitively, a τ-fragment of a process captures what occurs at the process during timeslot τ from the viewpoint of an omniscient external observer.

Given a τ-fragment $\mathcal{FR} = \left(s, \tau, \mathit{waiting}, M^R, T^R, M^S\right)$ of a process $p \in \Pi$ according to the protocol $\mathcal{P}$, let:

- $\mathcal{FR}.\mathsf{state} \equiv s$;
- $\mathcal{FR}.\mathsf{waiting} \equiv \mathit{waiting}$;
- $\mathcal{FR}.\mathsf{received\_msgs} \equiv M^R$;
- $\mathcal{FR}.\mathsf{received\_txs} \equiv T^R$;
- $\mathcal{FR}.\mathsf{sent\_msgs} \equiv M^S$;
- $\mathcal{FR}.\mathsf{quit} \equiv s.\mathsf{quit}$.

Behaviors. A tuple $\mathcal{B} = \left\langle \mathcal{FR}^0 = \left(s^0, 0, \mathit{waiting}^0, M^{R(0)}, T^{R(0)}, M^{S(0)}\right), ..., \mathcal{FR}^k = \left(s^k, k, \mathit{waiting}^k, M^{R(k)}, T^{R(k)}, M^{S(k)}\right) \right\rangle$ is a *($k \in \mathbb{N}_{\geq 0} \cup \{+\infty\}$)-long behavior* of a process $p \in \Pi$ according to the protocol $\mathcal{P}$ if and only if:

- for every $i \in [0, k]$, $\mathcal{FR}^i$ is an i-fragment of process p according to the protocol $\mathcal{P}$;
- there exists at most one transaction $tx \in \bigcup_{i \in [0,k]} T^{R(i)}$ with $tx.\mathsf{start} = \mathit{true}$;
- there exists at most one transaction $tx \in \bigcup_{i \in [0,k]} T^{R(i)}$ with $tx.\mathsf{quit} = \mathit{true}$.

Moreover, we say that $\mathcal{B}$ is a *valid* k-long behavior of process $p \in \Pi$ according to the protocol $\mathcal{P}$ if and only if:

- for every $i \in [0, k]$, $\mathcal{FR}^i$ is a valid i-fragment of process p;
- for every $i \in [0, k-1]$, $\mathsf{Transition}(s^i, M^{R(i)}, T^{R(i)}) = (s^{i+1}, M^{S(i)})$.

Given a k-long behavior $\mathcal{B} = \left\langle \mathcal{FR}^0, ..., \mathcal{FR}^k \right\rangle$ of a process $p \in \Pi$ according to the protocol $\mathcal{P}$, let:

- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{fragment}(i) \equiv \mathcal{FR}^i$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{state}(i) \equiv \mathcal{FR}^i.\mathsf{state}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{log}(i) = s^i.\mathsf{log}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{received\_msgs}(i) \equiv \mathcal{FR}^i.\mathsf{received\_msgs}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{received\_txs}(i) \equiv \mathcal{FR}^i.\mathsf{received\_txs}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{sent\_msgs}(i) \equiv \mathcal{FR}^i.\mathsf{sent\_msgs}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{quit}(i) \equiv \mathcal{FR}^i.\mathsf{quit}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{waiting}(i) \equiv \mathcal{FR}^i.\mathsf{waiting}$;
- for every $i \in [0, k]$, $\mathcal{B}.\mathsf{prefix}(i) \equiv \left\langle \mathcal{FR}^0, ..., \mathcal{FR}^i \right\rangle$.

Executions. A k-long execution $\mathcal{E}$ of the protocol $\mathcal{P}$, for any $k \in \mathbb{N}_{\geq 0} \cup \{+\infty\}$, is a tuple $[\mathcal{F}, \mathrm{GST} \in \mathsf{N}_{\geq 0}, \{\mathcal{B}_p \,|\, p \in \Pi\}]$ such that the following guarantees hold:

- *Faulty processes:* $\mathcal{F} \subseteq \Pi$ is a set of $|\mathcal{F}| \leq \rho \cdot n$ processes.
- *Composition:* For every process $p \in \Pi$, $\mathcal{B}_p$ is a k-long behavior of p according to the protocol $\mathcal{P}$.
- *Validity:* For every process $p \in \Pi \setminus \mathcal{F}$, $\mathcal{B}_p$ is a valid k-long behavior of p according to the protocol $\mathcal{P}$.
- *GST-validity:* For every process $p \in \Pi \setminus \mathcal{F}$ and every $i \in [\mathrm{GST}, k]$, the following holds: $\mathcal{B}_p.\mathsf{waiting}(i) = \mathit{false}$.
- *Receive-validity:* If there exists a message m, where $s = m.\mathsf{sender}$ and $r = m.\mathsf{receiver}$, such that $m \in \mathcal{B}_r.\mathsf{received\_msgs}(\tau_r)$, for some $\tau_r \in [0, k]$, then there exists $\tau_s \in [0, \tau_r - 1]$ such that $m \in \mathcal{B}_s.\mathsf{sent\_msgs}(\tau_s)$.
- *Send-validity:* Let there exist a message m, where $s = m.\mathsf{sender}$ and $r = m.\mathsf{receiver}$, such that $m \in \mathcal{B}_s.\mathsf{sent\_msgs}(\tau_s)$, for some $\tau_s \in [0, k]$. If $k \geq \max(\tau_s, \mathrm{GST}) + \delta$, then there exists $\tau_r \leq \max(\tau_s, \mathrm{GST}) + \delta$ such that $m \in \mathcal{B}_r.\mathsf{received\_msgs}(\tau_r)$.

B.3 Standard vs. Quit-Enhanced Blockchain Protocols

Finally, we are ready to formally define quit-enhanced blockchain protocols. To do so, we first provide a definition of standard blockchain protocols.

▶ **Definition 13** (Standard protocols)**.** *Fix any blockchain protocol* $\mathcal{P} = (\Pi, \rho,$ $\mathsf{States}, \{s_p^0 \,|\, p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}, \mathsf{Logs}, \mathsf{Transition})$*. We say that* $\mathcal{P}$ *is a* standard *blockchain protocol if and only if the following holds:*

- *for every state* $s \in \mathsf{States}$*,* $s.\mathsf{quit} = \mathit{false}$*;*
- *for every transaction* $tx \in \mathsf{Transactions}$*,* $tx.\mathsf{quit} = \mathit{false}$*.*

Intuitively, standard blockchain protocols do not have special quitting states and do not accept special quitting transactions from the environment. The following definition introduces quit-enhanced blockchain protocols.

▶ **Definition 14** (Quit-enhanced protocols)**.** *Fix any standard blockchain protocol* $\mathcal{P} = (\Pi, \rho, \mathsf{States}, \{s_p^0 \,|\, p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}, \mathsf{Logs}, \mathsf{Transition})$*. Let* $\mathcal{P}' = (\Pi, \rho, \mathsf{States}', \{s_p^0 \,|\, p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}', \mathsf{Logs}, \mathsf{Transition}')$ *be another blockchain protocol. We say that* $\mathcal{P}'$ *is the* quit-enhanced *version of* $\mathcal{P}$*, denoted by* $\mathit{quit}(\mathcal{P})$*, if and only if the following holds:*

- $\mathsf{States}' = \mathsf{States} \cup \{Q \,|\, (p \in \Pi, \mathcal{L} \in \mathsf{Logs}) : Q.\mathsf{process} = p \wedge Q.\mathsf{start} = \mathit{false} \wedge Q.\mathsf{quit} = \mathit{true} \wedge Q.\mathsf{log} = \mathcal{L}\}$*;*

- $\mathsf{Transactions}' = \mathsf{Transactions} \cup \{q \mid p \in \Pi : q.\mathsf{process} = p \wedge q.\mathsf{start} = \mathit{false} \wedge q.\mathsf{quit} = \mathit{true} \wedge q.\mathsf{log} = \bot\}$;
- *for every state* $s \in \mathsf{States}$ *(thus,* $s.\mathsf{quit} = \mathit{false}$*), every boolean waiting, every set of messages* $M^R \subseteq \mathsf{Messages}$ *and every set of transactions* $T^R \subseteq \mathsf{Transactions}$*, the following holds:*

$$\mathsf{Transition}'(s, \mathit{waiting}, M^R, T^R) = \mathsf{Transition}(s, \mathit{waiting}, M^R, T^R);$$

B.4 Preservation of Properties

Finally, this subsection proves that quit-enhanced blockchain protocols preserve key properties of their standard counterparts. This is crucial for proving the correctness of our transformation (cf. App. C). We fix any standard blockchain protocol $\mathcal{P} = (\Pi, \rho, \{s_p^0 \mid p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}, \mathsf{Logs}, \mathsf{Transition})$ and its quit-enhanced version $\mathcal{P}' = \mathit{quit}(\mathcal{P})$. We start by proving a crucial proposition used in showing that consistency and log-specific safety properties are translated from $\mathcal{P}$ to $\mathcal{P}'$.

▶ **Proposition 15.** *Let* $\mathcal{E}' = [\mathcal{F}', GST', \{\mathcal{B}'_p \mid p \in \Pi]\}]$ *be any* k*-long execution of* $\mathcal{P}'$*, for some* $k \in \mathbb{N}_{\geq 0}$ *(thus,* $k \neq +\infty$*). Then, there exists a* k*-long execution* $\mathcal{E} = [\mathcal{F}, GST, \{\mathcal{B}_p \mid p \in \Pi\}]$ *of* $\mathcal{P}$ *such that:*

- $\mathcal{F} = \mathcal{F}'$;
- *for every process* $p \in \mathcal{F}$ *and every* $i \in [0, k]$*, the following holds: (1)* $\mathcal{B}_p.\mathsf{state} \in \mathsf{States}$*, (2)* $\mathcal{B}_p.\mathsf{received\_msgs}(i) = \mathcal{B}'_p.\mathsf{received\_msgs}(i)$*, (3)* $\mathcal{B}_p.\mathsf{received\_txs}(i) = \mathcal{B}'_p.\mathsf{received\_txs}(i) \setminus (\mathsf{Transactions}' \setminus \mathsf{Transactions})$*, and (4)* $\mathcal{B}_p.\mathsf{sent\_msgs}(i) = \mathcal{B}'_p.\mathsf{sent\_msgs}(i)$*.*
- *for every process* $p \in \Pi \setminus \mathcal{F}$ *and every timeslot* $\tau \in [0, k]$*,* $\mathcal{B}_p.\mathsf{log}(\tau) = \mathcal{B}'_p.\mathsf{log}(\tau)$*.*

Proof. We divide all processes from the $\Pi \setminus \mathcal{F}'$ set into two disjoint groups:

- Let $Q' = \{p \in \Pi \setminus \mathcal{F}' \mid \exists i \in [0, k]\colon \mathcal{B}'_p.\mathsf{quit}(i) = \mathit{true}\}$.
- Let $NQ' = (\Pi \setminus \mathcal{F}') \setminus Q'$.

Step 1: Constructing a valid k*-long behavior* $\mathcal{B}_p$ *for a process* $p \in Q'$*.*
We construct $\mathcal{B}_p$ in the following way:

1. Let $i \in [0, k]$ denote the smallest integer such that $\mathcal{B}'_p.\mathsf{quit}(i) = \mathit{true}$.
2. For every $j \in [0, i-2]$, we set $\mathcal{B}_p.\mathsf{fragment}(j)$ to $\mathcal{B}'_p.\mathsf{fragment}(j)$.
3. Let $\mathcal{FR}^{i-1} = \big(\mathcal{B}'_p.\mathsf{fragment}(i-1).\mathsf{state}, i-1, \mathit{true}, \emptyset, \emptyset, \emptyset\big)$.
4. We set $\mathcal{B}_p.\mathsf{fragment}(i-1)$ to $\mathcal{FR}^{i-1}$.
5. For every $j \in [i, k]$, we set $\mathcal{B}'_p.\mathsf{fragment}(j)$ to $\big(\mathcal{FR}^{i-1}.\mathsf{state}, j, \mathit{true}, \emptyset, \emptyset, \emptyset\big)$.

Indeed, the constructed k-long behavior $\mathcal{B}_p$ is valid according to $\mathcal{P}$:

- Until (and including) timeslot $i-2$, $\mathcal{B}_p$ is valid as (1) it is identical to $\mathcal{B}'_p$ until (and including) timeslot $i-2$, and (2) $\mathcal{B}'_p$ is valid.
- We have that $\mathcal{B}_p.\mathsf{fragment}(i-1).\mathsf{state} = \mathcal{B}'_p.\mathsf{fragment}(i-1).\mathsf{state}$ because of the construction of $\mathcal{B}_p.\mathsf{fragment}(i-1)$. Hence, the transition from the $(i-2)$-nd to $(i-1)$-st fragment in $\mathcal{B}_p$ is valid according to the $\mathsf{Transition}$ function.
- For every timeslot $i \in [i-1, k]$, process p is waiting at timeslot i, thus not receiving any messages or transactions, not sending any messages, and not changing its state.

Moreover, for every timeslot $\tau \in [0, k]$, $\mathcal{B}_p.\mathsf{log}(\tau) = \mathcal{B}'_p.\mathsf{log}(\tau)$ due to (1) the definition of the quit-enhanced protocols, (2) the assumed constraints on the state transition function, and (3) our construction.

Step 2: Constructing a valid k*-long behavior* $\mathcal{B}_p$ *for a process* $p \in NQ'$*.*
We set $\mathcal{B}_p$ to $\mathcal{B}'_p$. Hence, $\mathcal{B}_p$ is trivially valid (as $\mathcal{B}'_p$ is) and, for every timeslot $\tau \in [0, k]$, $\mathcal{B}_p.\mathsf{log}(\tau) = \mathcal{B}'_p.\mathsf{log}(\tau)$.

Step 3: Constructing a k-long behavior $\mathcal{B}_p$ for a process $p \in \mathcal{F}'$.
We set $\mathcal{B}_p$ to $\mathcal{B}'_p$. Then, if there is a fragment of $\mathcal{B}_p$ in which p is in a state $s \in \mathsf{States}' \setminus \mathsf{States}$, we modify the fragment so that p is in any state from the States set. Similarly, if there is a fragment in which p receives any transaction $tx \in \mathsf{Transactions}' \setminus \mathsf{Transactions}$, we modify the fragment so that p does not receive this transaction.

Epilogue: Constructing $\mathcal{E}$.
Finally, let $\mathcal{E} = [\mathcal{F} = \mathcal{F}', k+1, \{\mathcal{B}_p \,|\, p \in \Pi\}]$. Note that the statement of the lemma is satisfied for $\mathcal{E}$. It is left to prove that $\mathcal{E}$ satisfies the guarantees of an execution:

- *Faulty processes:* Indeed, $|\mathcal{F}| \leq \rho \cdot n$ as $\mathcal{F} = \mathcal{F}'$ and $|\mathcal{F}'| \leq \rho \cdot n$.
- *Composition:* For every process $p \in \Pi$, $\mathcal{B}_p$ is a k-long behavior of p according to $\mathcal{P}$ (due to the construction of $\mathcal{B}_p$).
- *Validity:* For every process $p \in \Pi \setminus \mathcal{F}$, $\mathcal{B}_p$ is a valid k-long behavior of p (due to the construction of $\mathcal{B}_p$; see steps 1 and 2).
- *GST-validity:* As $k + 1 > k$, the guarantee is trivially satisfied.
- *Receive-validity:* First, note that if any process $p \in \Pi$ sends a message m in $\mathcal{B}'_p$ (i.e., $m \in \mathcal{B}'_p.\mathsf{sent\_msgs}$), then $\mathcal{B}_p$ sends m in $\mathcal{B}_p$ (i.e., $m \in \mathcal{B}_p.\mathsf{sent\_msgs}$). Let us distinguish three cases:
  - Let $p \in \mathcal{F}$. In this case, the statement is trivially true due to the construction of $\mathcal{B}_p$ (see step 3).
  - Let $p \in NQ'$. Again, the statement holds as $\mathcal{B}_p = \mathcal{B}'_p$ (see step 2).
  - Let $p \in Q'$. Let m be sent in the x-th fragment of $\mathcal{B}'_p$. Let $i \in [0, k]$ denote the smallest integer such that $\mathcal{B}'_p.\mathsf{quit} = \mathit{true}$. Note that $x \in [0, i-2]$ (due to the definition of the $\mathsf{Transition}'$ function). As $\mathcal{B}_p$ is identical to $\mathcal{B}'_p$ until (and including) timeslot $i-2$, m is sent in the x-th fragment of $\mathcal{B}_p$.

  Now, consider any message m received in $\mathcal{E}$. This implies that m is received in $\mathcal{E}'$, which then means that m is sent in $\mathcal{E}'$. Due to the argument above, m is sent in $\mathcal{E}$ as well, which proves the receive-validity property.
- *Send-validity:* As $k > k + 1$, the property trivially holds.

As $\mathcal{E}$ is indeed an execution of $\mathcal{P}$, the proof is concluded. ◀

We prove that the consistency property is translated from $\mathcal{P}$ to $\mathcal{P}'$.

▶ **Lemma 16** (Preservation of consistency)**.** *If $\mathcal{P}$ satisfies the consistency property, then $\mathcal{P}'$ satisfies the consistency property.*

Proof. By contradiction, suppose $\mathcal{P}'$ does not satisfy consistency. Let $\mathcal{E}' = [\mathcal{F}', \mathrm{GST}', \{\mathcal{B}'_p \,|\, p \in \Pi\}]$ be any k-long execution of $\mathcal{P}'$, for some $k \in \mathbb{N}_{\geq 0} \cup \{+\infty\}$, in which the consistency property is not satisfied. We study two cases:

- Let $k \in \mathbb{N}_{\geq 0}$ (thus, $k \neq +\infty$). By Prop. 15, there exists a k-long execution $\mathcal{E} = [\mathcal{F} = \mathcal{F}', \mathrm{GST}, \{\mathcal{B}_p \,|\, p \in \Pi\}]$ of $\mathcal{P}$ such that, for every process $p \in \Pi \setminus \mathcal{F}$ and every timeslot $\tau \in [0, k]$, $\mathcal{B}_p.\mathsf{log}(\tau) = \mathcal{B}'_p.\mathsf{log}(\tau)$. Hence, the consistency property is violated in $\mathcal{E}$, which represents a contradiction with the fact that $\mathcal{P}$ satisfies consistency.
- Let $k = +\infty$. We now define a *prefix*-long execution $\mathcal{E}'_{prefix}$ of $\mathcal{P}'$, for some $prefix \in \mathbb{N}_{\geq 0}$, in the following manner:
  - If there exist a process $p \in \Pi \setminus \mathcal{F}'$ and two timeslots $\tau_1, \tau_2 \in \mathbb{N}_{\geq 0}$ with $\tau_1 < \tau_2$ such that $\mathcal{B}'_p.\mathsf{log}(\tau_2)$ does not extend $\mathcal{B}'_p.\mathsf{log}(\tau_1)$, then let $\mathcal{E}'_{prefix} = [\mathcal{F}', \mathrm{GST}', \{\mathcal{B}^{prefix}_p \,|\, p \in \Pi\}]$, where, for each process $p \in \Pi$, $\mathcal{B}^{prefix}_p$ denotes the τ_2-long prefix of $\mathcal{B}'_p$ (i.e., $\mathcal{B}^{prefix}_p = \mathcal{B}'_p.\mathsf{prefix}(\tau_2)$).
  - Otherwise, there must exist a timeslot τ and a pair of processes $(p_1, p_2) \in (\Pi \setminus \mathcal{F}')^2$ such that logs $\mathcal{B}'_{p_1}.\mathsf{log}(\tau)$ and $\mathcal{B}'_{p_2}.\mathsf{log}(\tau)$ are not consistent. In this case, let $\mathcal{E}'_{prefix} =$

$[\mathcal{F}', \text{GST}', \{\mathcal{B}_p^{prefix} \mid p \in \Pi\}]$, where, for each process $p \in \Pi$, $\mathcal{B}_p^{prefix}$ denotes the τ-long prefix of $\mathcal{B}'_p$.

Note that the consistency property is violated in $\mathcal{E}'_{prefix}$. Finally, according to Prop. 15, there exists an execution of $\mathcal{P}$ that violates consistency. Given that $\mathcal{P}$ satisfies the property, this is impossible, which completes a contradiction in this case.

As the statement of the lemma holds in both cases, the proof is concluded. ◀

Next, we prove that log-specific safety properties are preserved in $\mathcal{P}'$.

▶ **Lemma 17** (Preservation of log-specific safety properties)**.** *If $\mathcal{P}$ satisfies any log-specific safety property S_L, then $\mathcal{P}'$ satisfies S_L.*

Proof. By contradiction, suppose $\mathcal{P}'$ does not satisfy the property S_L. Let $\mathcal{E}' = [\mathcal{F}', \text{GST}', \{\mathcal{B}'_p \mid p \in \Pi\}]$ be any k-long execution of $\mathcal{P}'$, for some $k \in \mathbb{N}_{\geq 0} \cup \{+\infty\}$, in which S_L is not satisfied. Let $\mathbb{L} = \bigcup_{i \in [0,k]} \bigcup_{p \in \Pi} \mathcal{B}'_p.\mathsf{log}(i)$. As S_L is not satisfied in $\mathcal{E}'$, we have that $S_L(\mathbb{L}) = \mathit{false}$.

We now consider two cases:

- Let $k \in \mathbb{N}_{\geq 0}$ (thus, $k \neq +\infty$). In this case, Prop. 15 proves that there exists an execution of $\mathcal{P}$ in which the property is not satisfied. As this is impossible, the statement of the lemma holds in this case.
- Let $k = +\infty$. As S_L is a log-specific safety property and $S_L(\mathbb{L}) = \mathit{false}$, there exists a finite subset $\mathit{logs}_{prefix} \subseteq \mathbb{L}$ such that $S_L(\mathit{logs}_{prefix}) = \mathit{false}$. Let τ denote the smallest timeslot such that, for every $\mathcal{L} \in \mathit{logs}_{prefix}$, there exist a process $\Pi \setminus \mathcal{F}'$ and a timeslot $\tau' \in [0, \tau]$ with $\mathcal{B}'_p.\mathsf{log}(\tau') = \mathcal{L}$. Now, let $\mathcal{E}'_{prefix} = [\mathcal{F}', \text{GST}', \{\mathcal{B}_p^{prefix} \mid p \in \Pi\}]$, where, for each process $p \in \Pi$, $\mathcal{B}_p^{prefix}$ denotes the τ-long prefix of $\mathcal{B}'_p$.
  Let $\mathbb{L}' = \bigcup_{i \in [0,\tau]} \bigcup_{p \in \Pi} \mathcal{B}_p^{prefix}.\mathsf{log}(i)$. Note that $\mathit{logs}_{prefix} \subseteq \mathbb{L}'$. Hence, $S_L(\mathbb{L}') = \mathit{false}$ according to the definition of log-specific safety properties. Finally, Prop. 15 proves that $\mathcal{P}$ violates S_L as well, thus concluding the proof in this case.

As the statement of the lemma holds in both cases, the proof is concluded. ◀

Finally, we prove a proposition required to prove liveness and optimistic responsiveness of our quasi-permissionless PoS protocol.

▶ **Proposition 18.** *Let $\mathcal{E}' = [\mathcal{F}', GST', \{\mathcal{B}'_p \mid p \in \Pi\}]$ be any k-long execution of $\mathcal{P}'$, for some $k \in \mathbb{N}_{\geq 0}$ (thus, $k \neq +\infty$), such that, for every $p \in \Pi \setminus \mathcal{F}'$ and every $i \in [0, k]$, $\mathcal{B}'_p.\mathsf{quit}(i) = \mathit{false}$ and $\mathcal{B}'_p.\mathsf{received\_txs}(i) \cap \mathsf{Transactions}' = \emptyset$. Then, there exists a k-long execution $\mathcal{E} = [\mathcal{F}, GST, \{\mathcal{B}_p \mid p \in \Pi\}]$ of $\mathcal{P}$ such that:*

- *$\mathcal{F} = \mathcal{F}'$;*
- *$GST = GST'$;*
- *for every process $p \in \mathcal{F}$ and every $i \in [0, k]$, the following holds: (1) $\mathcal{B}_p.\mathsf{state} \in \mathsf{States}$, (2) $\mathcal{B}_p.\mathsf{received\_msgs}(i) = \mathcal{B}'_p.\mathsf{received\_msgs}(i)$, (3) $\mathcal{B}_p.\mathsf{received\_txs}(i) = \mathcal{B}'_p.\mathsf{received\_txs}(i) \setminus (\mathsf{Transactions}' \setminus \mathsf{Transactions})$, and (4) $\mathcal{B}_p.\mathsf{sent\_msgs}(i) = \mathcal{B}'_p.\mathsf{sent\_msgs}(i)$.*
- *for every process $p \in \Pi \setminus \mathcal{F}$, $\mathcal{B}_p = \mathcal{B}'_p$.*

Proof. We construct $\mathcal{E}$ in the following manner.

Step 1: Constructing a k-long behavior $\mathcal{B}_p$ for a process $p \in \mathcal{F}'$.
We set $\mathcal{B}_p$ to $\mathcal{B}'_p$. Then, if there is a fragment of $\mathcal{B}_p$ in which p is in a state $s \in \mathsf{States}' \setminus \mathsf{States}$, we modify the fragment so that p is in any state from the States set. Similarly, if there is a fragment in which p receives any transaction $tx \in \mathsf{Transactions}' \setminus \mathsf{Transactions}$, we modify the fragment so that p does not receive this transaction.

Step 2: Constructing a valid k-long behavior $\mathcal{B}_p$ for a process $p \in \Pi \setminus \mathcal{F}'$. We set $\mathcal{B}_p$ to $\mathcal{B}'_p$.

Epilogue: Constructing $\mathcal{E}$.

Let $\mathcal{E} = [\mathcal{F} = \mathcal{F}', \text{GST} = \text{GST}', \{\mathcal{B}_p \,|\, p \in \Pi\}]$. The statement of the proposition is satisfied by $\mathcal{E}$. It is left to prove that $\mathcal{E}$ is indeed an execution of $\mathcal{P}$:

- *Faulty processes:* We have that $|\mathcal{F}| \leq \rho \cdot n$ as $\mathcal{F} = \mathcal{F}'$ and $|\mathcal{F}'| \leq \rho \cdot n$.
- *Composition:* For every process $p \in \Pi$, $\mathcal{B}_p$ is a k-long behavior of p according to $\mathcal{P}$.
- *Validity:* For every process $p \in \Pi \setminus \mathcal{F}$, $\mathcal{B}_p$ is a valid k-long behavior of p according to $\mathcal{P}$.
- *GST-validity:* This property is satisfied as (1) it is satisfied in $\mathcal{E}'$, and (2) for every process $p \in \Pi \setminus \mathcal{F}$, $\mathcal{B}_p = \mathcal{B}'_p$.
- *Receive-validity:* The set of messages sent (resp., received) in $\mathcal{E}$ is identical to the set of messages sent (resp., received) in $\mathcal{E}'$. Precisely, if a process p sends (resp., receives) a message a timeslot τ in $\mathcal{E}'$, then p sends (resp., receives) the message at timeslot τ in $\mathcal{E}$. Hence, the property holds in $\mathcal{E}$ as it holds in $\mathcal{E}'$.
- *Send-validity:* If a process p sends (resp., receives) a message a timeslot τ in $\mathcal{E}'$, then p sends (resp., receives) the message at timeslot τ in $\mathcal{E}$. Hence, the property holds in $\mathcal{E}$ as it holds in $\mathcal{E}'$.

Hence, $\mathcal{E}$ is an execution of $\mathcal{P}$, which concludes the proof. ◀

C Transformation: Omitted Definitions & Proof of Correctness

Throughout the entire section, we fix a permissioned protocol $\mathcal{P}$.

C.1 Omitted Definitions

In this subsection, we present definitions omitted from Sec. 4.2. We start by defining a method that returns the set of PoS-ids with positive stake.

▶ **Definition 19.** *Given any log $\mathcal{L}$, let* $\mathcal{L}.\mathsf{current\_ids} \equiv \{id \in \mathsf{IDs} \,|\, \mathsf{S}(\mathcal{L}, id) > 0\}$.

Given a log $\mathcal{L} = \mathcal{L}_\text{g}||[\mathsf{tr}_1, \mathsf{tr}_2, ..., \mathsf{tr}_x]$ with $x \geq 1$ transactions after the genesis log, let $\mathcal{L}.\mathsf{predecessor} = \mathcal{L}_\text{g}||[\mathsf{tr}_1, ..., \mathsf{tr}_{x-1}]$ be the prefix of $\mathcal{L}$ with $x - 1$ transactions. If $x = 1$, then $\mathcal{L}.\mathsf{predecessor} = \mathcal{L}_\text{g}$. Next, we define the epoch of a log. Importantly, the following definition relies on the completed method of a log; this method is defined later, in Def. 23. However, we underline that our definitions are not circular. Specifically, Defs. 20 to 23 should be considered one large recursive definition.

▶ **Definition 20.** *Given any log $\mathcal{L} \neq \mathcal{L}_\text{g}$ that extends $\mathcal{L}_\text{g}$, we define* $\mathcal{L}.\mathsf{epoch}$ *as:*

$$\mathcal{L}.\mathsf{epoch} \equiv \begin{cases} \mathcal{L}.\mathsf{predecessor.epoch}, & \text{if } \mathcal{L}.\mathsf{predecessor.completed} = \text{false} \\ \mathcal{L}.\mathsf{predecessor.epoch} + 1, & \text{if } \mathcal{L}.\mathsf{predecessor.completed} = \text{true}. \end{cases}$$

Moreover, $\mathcal{L}_\text{g}.\mathsf{epoch} \equiv 0$. *For any log $\mathcal{L}' \neq \mathcal{L}_\text{g}$ that does not extend $\mathcal{L}_\text{g}$,* $\mathcal{L}'.\mathsf{epoch} \equiv$ *undefined.*

Intuitively, if $\mathcal{L}.\mathsf{predecessor}$ is not a completed log (cf. Def. 23), then $\mathcal{L}$'s epoch is identical to the epoch of $\mathcal{L}.\mathsf{predecessor}$. Otherwise, $\mathcal{L}$ belongs to a new epoch. Next, we define the set of validators.

▶ **Definition 21.** *Given any log $\mathcal{L} \neq \mathcal{L}_\text{g}$ that extends $\mathcal{L}_\text{g}$, we define* $\mathcal{L}.\mathsf{validators}$ *as:*

$$\mathcal{L}.\mathsf{validators} \equiv \begin{cases} \mathcal{L}.\mathsf{predecessor.validators}, & \text{if } \mathcal{L}.\mathsf{predecessor.completed} = \text{false} \\ \mathcal{L}.\mathsf{predecessor.current\_ids}, & \text{if } \mathcal{L}.\mathsf{predecessor.completed} = \text{true}. \end{cases}$$

For any log $\mathcal{L}'$ such that (1) $\mathcal{L}'$ does not extend $\mathcal{L}_\text{g}$, or (2) $\mathcal{L}' = \mathcal{L}_\text{g}$, $\mathcal{L}'.\mathsf{validators} \equiv$ *undefined.*

Intuitively, $\mathcal{L}.\mathsf{validators}$ denotes the set of PoS-ids that produced $\mathcal{L}$. If $\mathcal{L}$ and $\mathcal{L}.\mathsf{predecessor}$ belong to the same epoch, their respective validator sets are identical. Otherwise, $\mathcal{L}.\mathsf{validators} = \mathcal{L}.\mathsf{predecessor}.\mathsf{current\_ids}$. Next, we define the concept of an epoch prefix of a log.

▶ **Definition 22.** *Given any log $\mathcal{L}$ that extends $\mathcal{L}_{\text{g}}$ and any $e \in [0, \mathcal{L}.\mathsf{epoch})$, let $\mathcal{L}.\mathsf{ep\_prefix}(e) \equiv \mathcal{L}'$, where (1) $\mathcal{L}$ extends $\mathcal{L}'$, (2) $\mathcal{L}'.\mathsf{epoch} = e$, and (3) $\mathcal{L}'.\mathsf{completed} = \mathit{true}$. For any $e \notin [0, \mathcal{L}.\mathsf{epoch})$, $\mathcal{L}.\mathsf{ep\_prefix}(e) \equiv \mathit{undefined}$.*

Given a log $\mathcal{L} \neq \mathcal{L}_{\text{g}}$ that extends $\mathcal{L}_{\text{g}}$, we define the $\mathcal{L}.\mathsf{finishers}$ set as:

$$\mathcal{L}.\mathsf{finishers} \equiv \{id \in \mathcal{L}.\mathsf{validators} \mid \exists (\text{FINISH}, \mathcal{L}.\mathsf{epoch}) \in \mathcal{L} \text{ issued by } id\}.$$

Moreover, given a log $\mathcal{L} \neq \mathcal{L}_{\text{g}}$ that extends $\mathcal{L}_{\text{g}}$, let us define $\mathcal{L}.\mathsf{can\_complete}$:

$$\mathcal{L}.\mathsf{can\_complete} \equiv \begin{cases} \mathit{true}, & \text{if } \sum_{id \in \mathcal{L}.\mathsf{finishers}} \mathsf{S}(\mathcal{L}.\mathsf{ep\_prefix}(\mathcal{L}.\mathsf{epoch} - 1), id) > \rho\mathbb{T} \\ \mathit{false}, & \text{otherwise.} \end{cases}$$

Now, we define what it means for a log to be completed.

▶ **Definition 23.** *Given any log $\mathcal{L} \neq \mathcal{L}_{\text{g}}$ that extends $\mathbb{T}$, $\mathcal{L}.\mathsf{completed} = \mathit{true}$ if and only if (1) $\mathcal{L}.\mathsf{can\_complete} = \mathit{true}$, and (2) there does not exist a log $\mathcal{L}'$ such that (a) $\mathcal{L}$ extends $\mathcal{L}'$, (b) $\mathcal{L}'.\mathsf{epoch} = \mathcal{L}.\mathsf{epoch}$, and (c) $\mathcal{L}'.\mathsf{can\_complete} = \mathit{true}$. Moreover, $\mathcal{L}_{\text{g}}.\mathsf{completed} = \mathit{true}$. For any log $\mathcal{L}' \neq \mathcal{L}_{\text{g}}$ that does not extend $\mathcal{L}_{\text{g}}$, $\mathcal{L}'.\mathsf{completed} \equiv \mathit{undefined}$.*

Next, we define certified logs.

▶ **Definition 24.** *A log $\mathcal{L} \neq \mathcal{L}_{\text{g}}$ that extends $\mathcal{L}_{\text{g}}$ is said to be* certified *at a process p if and only if p receives signatures on the log $\mathcal{L}$ from the members of a set $I \subseteq \mathcal{L}.\mathsf{validators}$ such that $\sum_{id \in I} \mathsf{S}(\mathcal{L}.\mathsf{ep\_prefix}(\mathcal{L}.\mathsf{epoch} - 1), id) \geq (1 - \rho)\mathbb{T}$. Moreover, $\mathcal{L}_{\text{g}}$ is certified at every process p.*

Lastly, we define fully-certified logs.

▶ **Definition 25.** *A certified log $\mathcal{L} \neq \mathcal{L}_{\text{g}}$ is said to be* fully-certified *at a process p if and only if $\forall e \in [0, \mathcal{L}.\mathsf{epoch})$, $\mathcal{L}.\mathsf{ep\_prefix}(e)$ is certified at process p_i. Moreover, $\mathcal{L}_{\text{g}}$ is fully-certified at every process p.*

C.2 Intermediate Propositions

In this section, we prove some intermediate propositions used throughout the entire section. For every log $\mathcal{L}$, let $\mathsf{processes}(\mathcal{L})$ be defined in the following way:

$$\mathsf{processes}(\mathcal{L}) \equiv \{p \in \Pi \mid \exists id \in \mathcal{L}.\mathsf{validators} : id \in \mathsf{id}(p)\}.$$

We underline that $\mathsf{processes}(\mathcal{L})$ is *not* known to processes (i.e., it is undetermined) as the identifier function $\mathsf{id}(\cdot)$ is undetermined. Next, we define *uncorrupted* logs.

▶ **Definition 26** (Uncorrupted logs)**.** *A log $\mathcal{L}$ is* uncorrupted *if and only if*

$$\sum_{p \in \mathcal{F} \cap \mathsf{processes}(\mathcal{L})} \sum_{id \in \mathsf{id}(p) \cap \mathcal{L}.\mathsf{validators}} \mathsf{S}(\mathcal{L}, id) \leq \rho \cdot \mathbb{T}.$$

We start by proving that, if $\mathsf{log}(p, \tau) = \mathcal{L}$, where p is a correct process and $\tau \in \mathbb{N}_{\geq 0}$ is a timeslot, then $\mathcal{L}.\mathsf{validators}$ is not overly corrupted.

▶ **Proposition 27.** *Consider any correct process p and any timeslot $\tau \in \mathbb{N}_{\geq 0}$. Then, every log $\mathcal{L}$ such that $\mathsf{log}(p, \tau)$ extends $\mathcal{L}$ is uncorrupted.*

Proof. The proposition holds because of the definition of the quasi-permissionless model (see App. A.3). ◀

Next, we prove that correct processes only adopt fully-certified logs.

▶ **Proposition 28.** *Consider any correct process p and any timeslot $\tau \in \mathbb{N}_{\geq 0}$. Then, $\mathsf{log}(p, \tau)$ is fully-certified at process p.*

Proof. The proposition trivially holds due to the rule at ln. 20. ◀

Next, we prove that correct processes only adopt logs that extend the genesis log $\mathcal{L}_{\mathrm{g}}$.

▶ **Proposition 29.** *Consider any correct process p and any timeslot $\tau \in \mathbb{N}_{\geq 0}$. Then, $\mathsf{log}(p, \tau)$ extends the genesis log $\mathcal{L}_{\mathrm{g}}$.*

Proof. The proposition trivially holds as (1) process p only extends its local log throughout every execution (due to the rule at ln. 20), and (2) $\mathsf{log}(p, 0) = \mathcal{L}_{\mathrm{g}}$. ◀

We say that a correct process p obtains a log $\mathcal{L}$ at a timeslot $\tau \in \mathbb{N}_{\geq 0}$ if and only if p receives an $\mathsf{obtain\_log}(\mathcal{L})$ event (see ln. 25) at timeslot τ.

▶ **Proposition 30.** *If any correct process p obtains a log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e > 0$, then the following holds for log $\mathcal{L}' = \mathcal{L}.\mathsf{ep\_prefix}(e-1)$: (1) $p \in \mathsf{processes}(\mathcal{L}')$, and (2) there exists a timeslot $\tau \in \mathbb{N}_{\geq 0}$ such that $\mathsf{log}(p, \tau) = \mathcal{L}'$.*

Proof. As process p obtains a log $\mathcal{L}$ (ln. 25), process p previously starts simulation of the underlying permissioned protocol $\mathcal{P}$ (ln. 19 or ln. 24) with some log $\mathcal{L}'$ and updates its local variable e_p to $\mathcal{L}'.\mathsf{epoch} + 1$. As p does obtain a log, that implies that $p \in \mathsf{processes}(\mathcal{L}')$. Moreover, when p starts simulation with $\mathcal{L}'$, it sets $\mathcal{L}'$ as the genesis log for the underlying permissioned protocol $\mathcal{P}$. As $\mathcal{P}$ satisfies consistency, any obtained log extends $\mathcal{L}'$. Finally, as p is only obtaining logs whose epoch is $e_p = \mathcal{L}'.\mathsf{epoch} + 1$, the statement of the proposition is satisfied for log $\mathcal{L}'$. ◀

C.3 Proof of Consistency

First, we prove that no correct process roll-backs its log.

▶ **Lemma 31** (No roll-backs)**.** *Consider any correct process p and any two timeslots $\tau_1, \tau_2 \in \mathbb{N}_{\geq 0}$ with $\tau_1 < \tau_2$. Then, $\mathsf{log}(p, \tau_2)$ extends $\mathsf{log}(p, \tau_1)$.*

Proof. The lemma trivially holds as p only updates its local log upon receiving a log that extends its current one (ln. 20). ◀

Next, we need to prove that the logs of correct processes never diverge. To this end, we define the set $\mathsf{logs}(e)$, for every epoch $e \in \mathbb{N}_{\geq 0}$:

$$\mathsf{logs}(e) = \{\mathcal{L} \mid \mathcal{L}.\mathsf{epoch} = e \wedge \exists (p \in \Pi, \tau \in \mathbb{N}_{\geq 0}) : p \text{ is correct} \wedge \mathsf{log}(p, \tau) = \mathcal{L}\}.$$

Note that $\mathsf{logs}(0) = \{\mathcal{L}_{\mathrm{g}}\}$. Now, we prove that only one completed log can exist within $\mathsf{logs}(e)$, for every epoch e, assuming that all logs in e are mutually consistent.

▶ **Proposition 32.** *Consider any epoch $e \in \mathbb{N}_{\geq 0}$. Suppose that for every pair of logs $(\mathcal{L}_1, \mathcal{L}_2) \in \big(\mathsf{logs}(e)\big)^2$, logs $\mathcal{L}_1$ and $\mathcal{L}_2$ are consistent. Then, there exists at most one log $\mathcal{L} \in \mathsf{logs}(e)$ such that $\mathcal{L}.\mathsf{completed} = \mathit{true}$.*

Proof. By contradiction, suppose at least two logs $\mathcal{L}_1$ and $\mathcal{L}_2$ exist such that (1) $\mathcal{L}_1 \in \mathsf{logs}(e)$, (2) $\mathcal{L}_2.\mathsf{logs}(e)$, and (3) $\mathcal{L}_1.\mathsf{completed} = \mathcal{L}_2.\mathsf{completed} = \mathit{true}$. As both logs belong to $\mathsf{logs}(e)$, they are consistent. However, it is then impossible that $\mathcal{L}_1.\mathsf{completed} = \mathcal{L}_2.\mathsf{completed} = \mathit{true}$ (due to Def. 23), which concludes the proof. ◀

Next, we prove some constraints on a process that obtains a specific log.

▶ **Proposition 33.** *If a correct process obtains a log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e > 0$, then $\mathcal{L}.\mathsf{ep\_prefix}(e-1) \in \mathsf{logs}(e-1)$.*

Proof. Recall that $\mathcal{L}.\mathsf{ep\_prefix}(e-1).\mathsf{epoch} = e - 1$. Hence, the proposition follows directly from Prop. 30. ◀

Next, we prove that every log $\mathcal{L} \in \mathsf{logs}(e)$ is obtained by a correct process.

▶ **Proposition 34.** *Consider any epoch $e \in \mathbb{N}$. For every log $\mathcal{L} \in \mathsf{logs}(e)$, $\mathcal{L}$ is obtained by a correct process.*

Proof. As $\mathcal{L} \in \mathsf{logs}(e)$, there exist a correct process p and a timeslot τ such that $\mathsf{log}(p, \tau) = \mathcal{L}$. Hence, $\mathcal{L}$ is fully-certified at process p (by Prop. 28). Moreover, $\mathcal{L}.\mathsf{ep\_prefix}(e-1)$ is uncorrupted (by Prop. 27). Therefore, there exists a correct process $q \in \mathsf{processes}(\mathcal{L}.\mathsf{ep\_prefix}(e-1))$ that obtains $\mathcal{L}$. Hence, the proposition holds. ◀

Next, we prove a direct consequence of Props. 33 and 34.

▶ **Proposition 35.** *If a correct process obtains a log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e > 0$, then, for every $e' \in [0, e-1]$, $\mathcal{L}.\mathsf{ep\_prefix}(e') \in \mathsf{logs}(e')$.*

Proof. The proposition is proven by (1) inductively applying Prop. 33 and then Prop. 34, and (2) the fact, for every epoch $e \in \mathbb{N}$ and every log $\mathcal{L} \in \mathsf{logs}(e)$, $\mathcal{L}_{\mathrm{g}} = \mathcal{L}.\mathsf{ep\_prefix}(0)$. ◀

Finally, we prove that all logs within $\mathsf{logs}(e)$ are mutually consistent, for every epoch $e \in \mathbb{N}_{\geq 0}$.

▶ **Proposition 36.** *Consider any epoch $e \in \mathbb{N}_{\geq 0}$ and any pair of logs $(\mathcal{L}_1, \mathcal{L}_2) \in \big(\mathsf{logs}(e)\big)^2$. Then, logs $\mathcal{L}_1$ and $\mathcal{L}_2$ are consistent.*

Proof. We prove the proposition by induction.

Base step: We prove the proposition holds for $e = 0$.
The proposition trivially holds for $e = 0$ as $\mathsf{logs}(0) = \{\mathcal{L}_{\mathrm{g}}\}$.

Inductive step: The proposition holds for some $e \geq 0$. We prove the proposition for $e + 1$.
Both logs $\mathcal{L}_1$ and $\mathcal{L}_2$ are obtained by correct processes (by Prop. 34). Hence, $\mathcal{L}_1.\mathsf{ep\_prefix}(e) \in \mathsf{logs}(e)$ and $\mathcal{L}_2.\mathsf{ep\_prefix}(e) \in \mathsf{logs}(e)$ (by Prop. 33). As $\mathcal{L}_1.\mathsf{ep\_prefix}(e).\mathsf{completed} = \mathcal{L}_2.\mathsf{ep\_prefix}(e).\mathsf{completed} = \mathit{true}$, $\mathcal{L}_1.\mathsf{ep\_prefix}(e) = \mathcal{L}_2.\mathsf{ep\_prefix}(e)$ (by Prop. 32).

Let r_1 be a correct process that obtains $\mathcal{L}_1$ and r_2 be a correct process that obtains $\mathcal{L}_2$. By Prop. 30, $p, q \in \mathsf{processes}(\mathcal{L}_1.\mathsf{ep\_prefix}(e))$. Therefore, logs $\mathcal{L}_1$ and $\mathcal{L}_2$ are indeed consistent as, by Lem. 16, the underlying protocol $\mathcal{P}$ satisfies consistency (even though correct processes might have stopped executing it, which goes against the standard permissioned model $\mathcal{P}$ is designed for). ◀

Finally, we prove that the logs of correct processes never diverge.

▶ **Lemma 37** (No divergence)**.** *Consider any pair of correct processes* $(p, q) \in \Pi^2$ *and any timeslot* $\tau \in \mathbb{N}_{\geq 0}$*. Then, logs* $\mathsf{log}(p, \tau)$ *and* $\mathsf{log}(q, \tau)$ *are consistent.*

Proof. Let $\mathcal{L}_p = \mathsf{log}(p, \tau)$ and $\mathcal{L}_q = \mathsf{log}(q, \tau)$. Moreover, let $e_p = \mathcal{L}_p.\mathsf{epoch}$ and $e_q = \mathcal{L}_q.\mathsf{epoch}$. Without loss of generality, assume $e_p \leq e_q$. We separate two cases:

- Let $e_p = e_q$. In this case, the lemma follows from Prop. 36.
- Let $e_p < e_q$. In this case, a correct process obtains $\mathcal{L}_q$ (by Prop. 34). Hence, $\mathcal{L}_q.\mathsf{prefix}(e_p) \in \mathsf{logs}(e_p)$ (by Prop. 35). Therefore, $\mathcal{L}_q.\mathsf{prefix}(e_p)$ and $\mathcal{L}_p$ are consistent (by Prop. 36), which implies that $\mathcal{L}_p$ and $\mathcal{L}_q$ are consistent.

The lemma holds as its statement is satisfied in both possible cases. ◀

C.4 Proof of Composable Log-Specific Safety Properties

Proof of Thm. 10. By Thm. 9, we know that $\mathcal{T}(\mathcal{P})$ satisfies consistency. We prove the lemma by induction and contradiction. Consider an execution $\mathcal{E}$ of $\mathcal{T}(\mathcal{P})$ and the set $\mathsf{logs} := \mathsf{logs}(\mathcal{E})$.

Base step: Given a ρ-bounded adversary, there exists an execution $\mathcal{E}$ of the quit-enhanced version $\mathcal{P}'$ of the protocol $\mathcal{P}$ such that $\mathsf{logs}(\mathcal{E}) = \mathsf{logs}(1)$. Then, since $\mathcal{P}$ satisfies P against a ρ-bounded static adversary, by Lem. 17, so does $\mathcal{P}'$, implying that $P(\mathsf{logs}(1)) = P(\mathsf{logs}(\mathcal{E})) =$ *true*. Hence, by Def. 6, for all $\mathsf{logs}' \subseteq \mathsf{logs}(1)$, it holds that $P(\mathsf{logs}') =$ *true*.

Inductive step: Let $\mathsf{logs}_e = \bigcup_{i=1,\ldots,e} \mathsf{logs}(e)$, and for all $\mathsf{logs}' \subseteq \mathsf{logs}_e$, it holds that $P(\mathsf{logs}') =$ *true*. By the safety of $\mathcal{T}(\mathcal{P})$, for any logs $\mathcal{L}, \mathcal{L}' \in \mathsf{logs}(e+1)$, it follows that $\mathcal{L}.\mathsf{prefix}(e) = \mathcal{L}'.\mathsf{prefix}(e)$, denoted by some $\mathcal{L}_e$. Thus, given a ρ-bounded adversary, there exists an execution $\mathcal{E}$ of the quit-enhanced version $\mathcal{P}'$ of the protocol $\mathcal{P}$ with the genesis log $\mathcal{L}_e$ such that $\mathsf{logs}(\mathcal{E}) = \mathsf{logs}(e+1)$. Then, since $\mathcal{P}$ satisfies P against a ρ-bounded static adversary, by Lem. 17, so does $\mathcal{P}'$, implying that $P(\mathsf{logs}(e+1)) = P(\mathsf{logs}(\mathcal{E})) =$ *true*. Therefore, by Def. 6, for all $\mathsf{logs}'' \subseteq \mathsf{logs}(e+1)$, it holds that $P(\mathsf{logs}'') =$ *true*. By Def. 7 (composability), this implies for any $\mathsf{logs}' \subseteq \mathsf{logs}_e$ and $\mathsf{logs}'' \subseteq \mathsf{logs}(e+1)$; for every $\mathsf{logs}^* \subseteq \mathsf{logs}' \cup \mathsf{logs}''$, $P(\mathsf{log}^*) =$ *true*. Therefore, for all $\mathsf{logs}^* \subseteq \mathsf{logs}_{e+1}$, it holds that $P(\mathsf{logs}') =$ *true*.

We have shown that for any $e < \infty$ and all $\mathsf{logs}^* \subseteq \mathsf{logs}_{e+1}$, $P(\mathsf{logs}^*) =$ *true*. Finally, suppose $P(\mathsf{logs}) =$ *false*. Then, by Def. 6, there exists a finite subset $\mathsf{logs}^* \subseteq \mathsf{logs}$ such that $P(\mathsf{logs}^*) =$ *false*. Now, let $e_{max} < \infty$ denote the largest epoch among the logs in logs^*, i.e. $e_{max} = \max\{\mathcal{L}.\mathsf{epoch} | \mathcal{L} \in \mathsf{logs}^*\}$, which is well-defined since logs^* is finite. However, then $\mathsf{logs}^* \subseteq \mathsf{logs}_{e_{max}}$ and for all $\mathsf{logs}^* \subseteq \mathsf{logs}_{e_{max}}$, we know that $P(\mathsf{logs}') =$ *true*, as $e_{max} < \infty$. This is a contradiction, which implies $P(\mathsf{logs}) =$ *true*. ◀

C.5 Proof of Liveness

Suppose the adversary is a ρ-bounded static adversary in the quasi-permissionless setting. Recall that logs denote the set of logs, including their prefixes, output by correct processes. By Thm. 9, we observe that the protocol $\mathcal{T}(\mathcal{P})$ is ρ-consistent.

▶ **Lemma 38.** *For every epoch* $e \in \mathbb{N}$*, there exists a unique log* $\mathcal{L}^*(e)$ *such that (1)* $\mathcal{L}^*(e) \in \mathsf{logs}$*, (2)* $\mathcal{L}^*(e).\mathsf{epoch} = e$*, and (3)* $\mathcal{L}^*(e).\mathsf{completed} =$ *true. Moreover, for any* $e < e'$*,* $\mathcal{L}^*(e')$ *extends* $\mathcal{L}^*(e)$*.*

Proof. By the consistency of $\mathcal{T}(\mathcal{S})$, all logs obtained by the correct processes are consistent. Therefore, for any two logs $\mathcal{L}$ and $\mathcal{L}'$ obtained by the correct processes, if $\mathcal{L}.\mathsf{ep\_prefix}(e)$ and

$\mathcal{L}'.\mathsf{ep\_prefix}(e)$ are both defined, it holds that $\mathcal{L}.\mathsf{ep\_prefix}(e) = \mathcal{L}'.\mathsf{ep\_prefix}(e)$ by property (1), $\mathcal{L}.\mathsf{epoch} = \mathcal{L}'.\mathsf{epoch} = e$ by property (2), and $\mathcal{L}.\mathsf{completed} = \mathcal{L}'.\mathsf{completed} = \mathit{true}$ by property (3) in Def. 22, all of which imply the first statement of the lemma. Moreover, by consistency and Def. 20, for any $e < e'$, $\mathcal{L}^*(e')$ extends $\mathcal{L}^*(e)$. ◀

We say that a process p is an e-process if there exists an identifier $id \in \mathsf{id}(p)$ such that $id \in \mathcal{L}.\mathsf{validators}$, where the $\mathcal{L} \in \mathsf{logs}(e)$. For each epoch $e \in \mathbb{N}_{\geq 1}$, let $\tau(e)$ denote the timeslot at which the first correct e-process enters epoch e at ln. 19 or ln. 24. Formally, $\tau(e)$ is the first timeslot a correct process holds a log $\mathcal{L}$ such that $\mathcal{L}.\mathsf{epoch} = e - 1$ and $\mathcal{L}.\mathsf{completed} = \mathit{true}$. Lastly, we say that an epoch $e \in \mathbb{N}_{\geq 1}$ is a *post-GST epoch* if $\tau(e) \geq \text{GST}$. Define $\mathsf{ED} = \ell + \Delta < \infty$.

▶ **Lemma 39.** *Let $e \in \mathbb{N}_{\geq 1}$ and $e' \in \mathbb{N}_{\geq 1}$ be any two post-GST epochs such that $e < e'$. Then, $\tau(e') \geq \tau(e) + \mathsf{ED}$.*

Proof. By Lem. 38, $\mathcal{L}^\star(e')$ extends $\mathcal{L}^\star(e)$. By the fact that $\mathcal{L}^\star(e) \in \mathsf{logs}$ and Prop. 28, the log $\mathcal{L}^\star(e)$ is fully-certified (ln. 19). Moreover, for any $\tilde{e} \in [0, e']$, by Prop. 27,

$$\sum_{\text{corrupt } id \in \mathcal{L}^\star(\tilde{e}).\mathsf{validators}} \mathsf{S}(\mathcal{L}^\star(\tilde{e}), id) \leq \rho \cdot \mathcal{L}^\star(\tilde{e}).\mathsf{total\_stake}$$

A correct e'-process p_i receives a fully-certified log $\mathcal{L}^\star(e'-1)$ (ln. 19) before entering epoch e' (ln. 19 and ln. 24). Therefore, as $1 - \rho > \rho$, there exists a correct $(e'-1)$-process p_j that signed the log $\mathcal{L}^\star(e'-1)$ by timeslot $\tau(e')$, and by Def. 23, there is a transaction $(\textsc{finish}, e)$ on $\mathcal{L}^\star(e'-1)$ that was sent by a correct $(e'-1)$-process. Since no correct $(e'-1)$-process sends such a transaction before timeslot $\tau(e'-1) + \mathsf{ED}$ and as $\tau(e_1) \geq \tau(e_2)$ for any $e_1 \geq e_2$ by Alg. 1, it holds that no correct $(e'-1)$-process sends such a transaction before timeslot $\tau(e) + \mathsf{ED}$, implying that $\tau(e') \geq \tau(e) + \mathsf{ED}$. ◀

Next, we prove that, given any post-GST epoch e, all correct e-processs enter epoch e by timeslot $\tau(e) + \delta$, where δ is the actual message delay.

▶ **Lemma 40.** *Let $e \in \mathbb{N}_{\geq 1}$ be any post-GST epoch. Then, all correct e-processs enter epoch e by timeslot $\tau(e) + \delta$.*

Proof. If $e = 1$, the lemma trivially holds as all correct 1-processes start executing the protocol at time 0, and $\text{GST} = 0$. Hence, let $e > 1$.

The first correct e-process enters epoch e at timeslot $\tau(e)$ upon receiving a fully-certified log $\mathcal{L}^\star(e-1)$. As $\tau(e) \geq \text{GST}$, all correct e-processs receive $\mathcal{L}^\star(e-1)$ by timeslot $\tau(e) + \delta$. Let $\mathcal{L}'$ denote the log of a process p_j right before the timeslot it received $\mathcal{L}^\star(e-1)$. Either $\mathcal{L}'.\mathsf{epoch} \leq e$, which implies p_j enters epoch e by timeslot $\tau(e) + \delta$, or $\mathcal{L}'.\mathsf{epoch} = e' > e$, which implies $\tau(e') \leq \tau(e) + \delta$ and contradicts with Lem. 39 Therefore, all correct e-processes enter epoch e by timeslot $\tau(e) + \delta$. ◀

Proof of Thm. 11. Let tr be a transaction received by a correct process at some timeslot τ for the first time. Let e_{max} denote the largest epoch entered by an honest process by timeslot $\max(\tau, \text{GST}) + \delta$, i.e.,

$$e_{max} = \max_{\mathcal{L} \in \mathsf{logs} \text{ at } \max(\tau, \text{GST}) + \delta} \mathcal{L}.\mathsf{epoch}$$

Importantly, every correct e_{max}-process p_i obtains tr by timeslot $\max(\tau, \text{GST}) + \delta$. We distinguish the following two cases:

- Suppose $\tau(e_{max}+1) > \max(\tau, \mathrm{GST}) + \delta + \ell$. By Lem. 40, all correct e_{max}-processes enter epoch e_{max} by timeslot $\max(\tau, \mathrm{GST}) + \delta \geq \tau(e_{max}) + \delta$, by which they receive tr, and after timeslot $\max(\tau, \mathrm{GST}) + \delta$, they all stay in epoch e_{max} for at least ℓ more timeslots (all after GST). Therefore, by Prop. 18 and the (ρ, ℓ)-liveness of the protocol $\mathcal{P}$, every correct e_{max}-process obtains some log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e_{max}$ containing tr by timeslot $\max(\tau, \mathrm{GST}) + \delta + \ell$[7]. Since for all logs $\mathcal{L}$, $\mathcal{L}.\mathsf{epoch} > e_{max}$, obtained by a correct process it holds that $\mathcal{L}.\mathsf{ep\_prefix}(e_{max}) \in \mathsf{logs}(e_{max})$ by Prop. 33, for any correct process that is active (and non-waiting) at a timeslot $\tau_{\mathrm{a}} \geq \max(\tau, \mathrm{GST}) + \delta + \ell$, i.e., for any epoch e-process, $e \geq e_{max}$, it holds that $\mathsf{tr} \in \mathsf{log}(p, \tau_{\mathrm{a}})$.

  Moreover, if $\mathcal{P}$ is also $(\rho, \ell_{\mathrm{or}})$-responsive, every correct e_{max}-process obtains some log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e_{max}$ containing tr by timeslot $\max(\tau, \mathrm{GST}) + \delta + \ell_{\mathrm{or}}$, where $\ell_{\mathrm{or}} \in O(\delta)$. Since for all logs $\mathcal{L}$, $\mathcal{L}.\mathsf{epoch} > e_{max}$, obtained by a correct process it holds that $\mathcal{L}.\mathsf{ep\_prefix}(e_{max}) \in \mathsf{logs}(e_{max})$ by Prop. 33, for any correct process that is active (and non-waiting) at a timeslot $\tau_{\mathrm{a}} \geq \max(\tau, \mathrm{GST}) + \delta + \ell_{\mathrm{or}}$, i.e., for any epoch e-process, $e \geq e_{max}$, it holds that $\mathsf{tr} \in \mathsf{log}(p, \tau_{\mathrm{a}})$.
- Let $\tau(e_{max}+1) \leq \max(\tau, \mathrm{GST}) + \delta + \ell$. In this case, by Lem. 40, every correct $(e_{max}+1)$-process receives tr and enters epoch $\tau(e_{max}+1)$ by timeslot $\tau(e_{max}+1) + \delta > \mathrm{GST}$, and by Lem. 39, after timeslot $\tau(e_{max}+1) + \delta$, they all stay in epoch $e_{max}+1$ for at least ℓ more timeslots as $\mathsf{ED} = \ell + \Delta > \ell + \delta$ (all after GST). Therefore, by Prop. 18 and the (ρ, ℓ)-liveness of the protocol $\mathcal{P}$, every correct $(e_{max}+1)$-process obtains a log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e_{max}+1$ containing tr by timeslot $\tau(e_{max}+1) + \ell + \delta \leq \max(\tau, \mathrm{GST}) + 2\delta + 2\ell$. Since for all logs $\mathcal{L}$, $\mathcal{L}.\mathsf{epoch} > e_{max}+1$, obtained by a correct process it holds that $\mathcal{L}.\mathsf{ep\_prefix}(e_{max}+1) \in \mathsf{logs}(e_{max}+1)$ by Prop. 33, for any correct process that is active (and non-waiting) at a timeslot $\tau_{\mathrm{a}} \geq \max(\tau, \mathrm{GST}) + 2\delta + 2\ell$, i.e., for any epoch e-process, $e \geq e_{max}+1$, it holds that $\mathsf{tr} \in \mathsf{log}(p, \tau_{\mathrm{a}})$.

  Now, suppose $\mathcal{P}$ is also $(\rho, \ell_{\mathrm{or}})$-responsive, and consider the following two cases:

  - Suppose $\tau(e_{max}+1) > \max(\tau, \mathrm{GST}) + \delta + \ell_{\mathrm{or}}$. Then, by Lem. 40, all correct e_{max}-processes enter epoch e_{max} by timeslot $\max(\tau, \mathrm{GST}) + \delta \geq \tau(e_{max}) + \delta$, by which they receive tr, and after timeslot $\max(\tau, \mathrm{GST}) + \delta$, they all stay in epoch e_{max} for at least ℓ_{or} more timeslots (all after GST). Therefore, by the $(\rho, \ell_{\mathrm{or}})$-responsiveness of the protocol $\mathcal{P}$, every correct e_{max}-process obtains some log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e_{max}$ containing tr by timeslot $\max(\tau, \mathrm{GST}) + \delta + \ell_{\mathrm{or}}$. Since for all logs $\mathcal{L}$, $\mathcal{L}.\mathsf{epoch} > e_{max}$, obtained by a correct process it holds that $\mathcal{L}.\mathsf{ep\_prefix}(e_{max}) \in \mathsf{logs}(e_{max})$ by Prop. 33, for any correct process that is active (and non-waiting) at a timeslot $\tau_{\mathrm{a}} \geq \max(\tau, \mathrm{GST}) + \delta + \ell_{\mathrm{or}}$, i.e., for any epoch e-process, $e \geq e_{max}$, it holds that $\mathsf{tr} \in \mathsf{log}(p, \tau_{\mathrm{a}})$.
  - Suppose $\tau(e_{max}+1) \leq \max(\tau, \mathrm{GST}) + \delta + \ell_{\mathrm{or}}$. In this case, by Lem. 40, every correct $(e_{max}+1)$-process receives tr and enters epoch $\tau(e_{max}+1)$ by timeslot $\tau(e_{max}+1) + \delta > \mathrm{GST}$, and by Lem. 39, after timeslot $\tau(e_{max}+1) + \delta$, they all stay in epoch $e_{max}+1$ for at least ℓ more timeslots as $\mathsf{ED} = \ell + \Delta > \ell + \delta$ (all after GST). Therefore, by the $(\rho, \ell_{\mathrm{or}})$-responsiveness of the protocol $\mathcal{P}$, every correct $(e_{max}+1)$-process obtains a log $\mathcal{L}$ with $\mathcal{L}.\mathsf{epoch} = e_{max}+1$ containing tr by timeslot $\tau(e_{max}+1) + \ell_{\mathrm{or}} + \delta \leq \max(\tau, \mathrm{GST}) + 2\delta + \ell_{\mathrm{or}}$. Since for all logs $\mathcal{L}$, $\mathcal{L}.\mathsf{epoch} > e_{max}+1$, obtained by a correct process it holds that $\mathcal{L}.\mathsf{ep\_prefix}(e_{max}+1) \in \mathsf{logs}(e_{max}+1)$ by Prop. 33, for any correct

[7] Even though the correct processes stop executing the protocol at the end of the epoch, which goes against the standard permissioned model $\mathcal{P}$ is designed for, by Prop. 18, we know that the liveness of $\mathcal{P}$ carries over to the liveness of its quit-enhanced version.

process that is active (and non-waiting) at a timeslot $\tau_a \geq \max(\tau, \text{GST}) + 2\delta + 2\ell_{\text{or}}$, i.e., for any epoch e-process, $e \geq e_{max} + 1$, it holds that $\mathsf{tr} \in \mathsf{log}(p, \tau_a)$.

This concludes the theorem. ◀

C.6 Proof of Accountability

In this subsection, we prove Thm. 12.

Proof of Thm. 12. Suppose there exist correct processes $p \in \Pi$ and $q \in \Pi$, and timeslots $\tau_p \in \mathbb{N}_{\geq 1}$ and $\tau_q \in \mathbb{N}_{\geq 1}$ such that $\mathsf{log}(p, \tau_p)$ is inconsistent with $\mathsf{log}(q, \tau_q)$. Let L denote the longest log (i.e., the log with the most transactions) such that (1) $\mathsf{log}(p, \tau_p)$ extends L, and (2) $\mathsf{log}(q, \tau_q)$ extends L. Note that such log L exists as p and q share the same genesis log $\mathcal{L}_g$. Moreover, note that $L.\mathsf{length} < \mathsf{log}(p, \tau_p).\mathsf{length}$ and $L.\mathsf{length} < \mathsf{log}(q, \tau_q).\mathsf{length}$.

Let L_p denote the log such that (1) $L_p.\mathsf{predecessor} = L$, and (2) $\mathsf{log}(p, \tau_p)$ extends L_p. Similarly, let L_q denote the log such that (1) $L_q.\mathsf{predecessor} = L$, and (2) $\mathsf{log}(q, \tau_q)$ extends L_q. By the definition of L, L_p, and L_q, logs L_p and L_q are inconsistent. Crucially, $L_p.\mathsf{epoch} = L_q.\mathsf{epoch}$ and $L_p.\mathsf{validators} = L_q.\mathsf{validators}$ by Def. 21, since both logs share the same predecessor log L, i.e., $L_p.\mathsf{predecessor} = L_q.\mathsf{predecessor} = L$.

Moreover, let us define the log $L_p^\star$ in the following way:

$$L_p^\star \equiv \begin{cases} \mathsf{log}(p, \tau_p), & \text{if } \mathsf{log}(p, \tau_p).\mathsf{epoch} = L_p.\mathsf{epoch} \\ \mathsf{log}(p, \tau_p).\mathsf{ep\_prefix}(L_p.\mathsf{epoch}), & \text{otherwise.} \end{cases}$$

We define the log $L_q^\star$ in the same way:

$$L_q^\star \equiv \begin{cases} \mathsf{log}(q, \tau_q), & \text{if } \mathsf{log}(q, \tau_q).\mathsf{epoch} = L_q.\mathsf{epoch} \\ \mathsf{log}(q, \tau_q).\mathsf{ep\_prefix}(L_q.\mathsf{epoch}), & \text{otherwise.} \end{cases}$$

Note that (1) $L_p^\star.\mathsf{epoch} = L_q^\star.\mathsf{epoch} = L_p.\mathsf{epoch} = L_q.\mathsf{epoch}$, and (2) $L_p^\star.\mathsf{validators} = L_q^\star.\mathsf{validators} = L_p.\mathsf{validators} = L_q.\mathsf{validators}$. Let $\mathbb{V} = L_p^\star.\mathsf{validators} = L_q^\star.\mathsf{validators}$. Moreover, $L_p^\star$ extends L_p and $L_q^\star$ extends L_q, which implies that $L_p^\star$ and $L_q^\star$ are inconsistent.

Finally, observe that since processes p and q have received fully certified logs $\mathsf{log}(p, \tau_p)$ and $\mathsf{log}(q, \tau_q)$, it follows that: (1) by timeslot τ_p, process p obtained a quorum (with respect to $\mathbb{V}$) of signatures on $L_p^\star$, and (2) by timeslot τ_q, process q obtained a quorum (with respect to $\mathbb{V}$) of signatures on $L_q^\star$. Therefore, the messages received by p and q when collecting signatures on logs $L_p^\star$ and $L_q^\star$, respectively, contain proof of guilt of at least $(1-2\rho)\mathbb{T}$-worth of identifiers in $\mathbb{V}$. This follows from the fact that each process received signatures from $(1-\rho)\mathbb{T}$-worth of stake. Thus, the quasi-permissionless protocol $\mathcal{T}(\mathcal{P})$ satisfies $(1-2\rho)$-accountability. ◀

C.7 Proof of Message Complexity

In this subsection, we study the message complexity of a quasi-permissionless PoS protocol $\mathcal{T}(\mathcal{P})$ obtained by transforming a permissioned protocol $\mathcal{P}$. When studying message complexity, one can focus on two distinct aspects:

- *Within-epoch messages:* messages exchanged between validators within each epoch.
- *Outside-epoch messages:* messages exchanged between non-validator processes within each epoch.

When it comes to within-epoch messages, our transformation adds an additional "signature-round" to ensure accountability. Hence, our transformation adds a quadratic additive factor to the number of within-epoch messages sent in the permissioned protocol $\mathcal{P}$.

Regarding the outside-epoch messages, in our transformation, processes continue to disseminate their logs to all processes, not just validators. This is essential because all processes, including non-validators, must finalize new transactions. However, as even permissioned protocols must account for this type of dissemination (assuming they serve an undetermined universe of "clients"), our definition of message complexity focuses exclusively on the messages exchanged among validators (i.e., on the number of exchanged within-epoch messages).

Formally, consider a standard blockchain protocol $\mathcal{P}$

$$(\Pi, \rho, \mathsf{States}, \{s_p^0 \mid p \in \Pi\}, \mathsf{Messages}, \mathsf{Transactions}, \mathsf{Logs}, \mathsf{Transition}),$$

with the set of processes Π such that $|\Pi| = n$. Let the tuple $\mathcal{B} = \left\langle \mathcal{FR}_{\mathcal{E},p}^0, \ldots, \mathcal{FR}_{\mathcal{E},p}^k = \left(s_{\mathcal{E},p}^k, k, \mathit{waiting}_{\mathcal{E},p}^k, M_{\mathcal{E},p}^{R(k)}, T_{\mathcal{E},p}^{R(k)}, M_{\mathcal{E},p}^{S(k)}\right) \right\rangle$ be a k-long behavior of a process $p \in \Pi$ according to an execution $\mathcal{E}$ of the protocol $\mathcal{P}$, where $T_{\mathcal{E},p}^{R(k)}$, $M_{\mathcal{E},p}^{S(k)}$ and $M_{\mathcal{E},p}^{R(k)}$ respectively denote the transactions received, and the messages sent and received by p at timeslot k. By $|T_{\mathcal{E},p}^{R(k)}|$, we denote the sums of the lengths of the transactions measured in bits within the set $T_{\mathcal{E},p}^{R(k)}$, and $|M_{\mathcal{E},p}^{S(k)}|$ and $|M_{\mathcal{E},p}^{R(k)}|$ are defined similarly.

▶ **Definition 41.** *We define the* average message complexity *of $\mathcal{P}$ under ρ-bounded static adversaries, denoted by* AC*, as follows:*

$$\mathsf{AC} = \lim_{j \to \infty} \sup_{\mathcal{E} \in E} \frac{1}{j} \sum_{p \in \Pi \setminus \mathcal{F}} \sum_{i=0}^{j} (|T_{\mathcal{E},p}^{R(i)}| + |M_{\mathcal{E},p}^{S(i)}| + |M_{\mathcal{E},p}^{R(i)}|),$$

where E denotes the set of executions of the protocol $\mathcal{P}$ under all ρ-bounded static adversaries.

▶ **Definition 42.** *We define the* peak message complexity *of $\mathcal{P}$, denoted by* PC *as follows:*

$$\mathsf{PC} = \sup_{\mathcal{E} \in E} \sup_{i \in [0,\infty), p \in \Pi \setminus \mathcal{F}} (|T_{\mathcal{E},p}^{R(i)}| + |M_{\mathcal{E},p}^{S(i)}| + |M_{\mathcal{E},p}^{R(i)}|),$$

where E denotes the set of executions of the protocol $\mathcal{P}$ under all ρ-bounded static adversary.

We assume that all expressions within the limits above are continuous functions of n and converge uniformly to a function of n, and AC, PC are finite.

Given these definitions, we can state our theorem on the message complexity of $\mathcal{T}(\mathcal{P})$. We assume that $\mathrm{GST} = 0$ in the following analysis to capture the communication complexity of the protocols while they finalize new transactions.

▶ **Theorem 43.** *Consider a permissioned protocol $\mathcal{P}$ that has finite average and peak message complexity of $\mathsf{AC} = \Omega(n^2)$ and $\mathsf{PC} = \Omega(n^2)$ against ρ-bounded static adversaries. Then, there exist a parameter ℓ such that $\mathcal{T}(\mathcal{P})$ with parameter ℓ has average and peak message complexity of $\mathsf{AC}' = O(\mathsf{AC})$ and $\mathsf{PC} = O(\mathsf{AC}')$ against ρ-bounded static adversaries.*

Intuitively, Thm. 43 states that the message complexities are asymptotically the same for both the permissioned and PoS protocols. It follows from the fact that the PoS protocol simply runs multiple iterations of the permissioned protocol stitched together, with quadratic overhead in the number of processes (e.g., epoch-ending messages), which is absorbed by the super-quadratic asymptotics of the permissioned protocol.

Proof. Let $\tilde{T}_{\mathcal{E},p}^{R(i)}$, $\tilde{M}_{\mathcal{E},p}^{S(i)}$, $\tilde{M}_{\mathcal{E},p}^{R(i)}$, $\tilde{\mathsf{AC}}$, $\tilde{\Pi}$ and $\tilde{E}$ denote the same definitions on the PoS protocol $\mathcal{T}(\mathcal{P})$, and suppose both protocols have infinite running times. By the definition of AC, for

any positive $\epsilon < 1$, there exists an $\ell > 0$ such that

$$\sup_{\mathcal{E} \in E} \frac{1}{2\ell} \sum_{p \in \Pi \setminus \mathcal{F}} \sum_{i=0}^{2\ell} (|T_{\mathcal{E},p}^{R(i)}| + |M_{\mathcal{E},p}^{S(i)}| + |M_{\mathcal{E},p}^{R(i)}|) \leq (1+\epsilon)\mathsf{AC}$$

Consider $\mathcal{T}(\mathcal{P})$ instantiated with this parameter ℓ. Then, counting the epoch-ending transactions and the transactions $\mathsf{tr} = \{q \,|\, p \in \Pi : q.\mathsf{process} = p \wedge q.\mathsf{start} = \mathit{false} \wedge q.\mathsf{quit} = \mathit{true} \wedge q.\mathsf{log} = \bot\}$, both of finite size, we can state that for $\mathcal{T}(\mathcal{P})$ and $j = r\ell$;

$$\begin{aligned}
&\sup_{\tilde{\mathcal{E}} \in \tilde{E}} \frac{1}{j} \sum_{p \in \tilde{\Pi} \setminus \mathcal{F}} \sum_{i=0}^{j} (|\tilde{T}_{\tilde{\mathcal{E}},p}^{R(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{S(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{R(i)}|) \\
\leq &\sup_{\mathcal{E}_1, \ldots \mathcal{E}_r \in E} \frac{2}{r} \sum_{i=1}^{r} \frac{1}{2\ell} \sum_{p \in \Pi \setminus \mathcal{F}} \sum_{i=0}^{2\ell} (|T_{\mathcal{E}_i,p}^{R(i)}| + |M_{\mathcal{E}_i,p}^{S(i)}| + |M_{\mathcal{E}_i,p}^{R(i)}|) \\
\leq &\frac{2}{r} \sum_{i=1}^{r} \sup_{\mathcal{E}_i \in E} \frac{1}{\ell} \sum_{p \in \Pi \setminus \mathcal{F}} \sum_{i=0}^{2\ell} (|T_{\mathcal{E}_i,p}^{R(i)}| + |M_{\mathcal{E}_i,p}^{S(i)}| + |M_{\mathcal{E}_i,p}^{R(i)}|) \\
\leq &n^2 C + \frac{2}{r} \sum_{i=1}^{r} (1+\epsilon)\mathsf{AC},
\end{aligned}$$

for some executions $\mathcal{E}_i$ of $\mathcal{P}$ under a ρ-bounded adversary. Here, C is some constant finite value capturing the size of the epoch-ending transactions and the transactions tr mentioned above. There are n^2 number of them; since n identifiers send these transactions once, and they are re-broadcast to n identifiers from n identifiers once more as part of the fully-certified logs. Moreover, note that we consider a duration 2ℓ execution of the permissioned protocol in our bound; since no correct process will stay in an epoch longer than 2ℓ (in fact, $\ell + O(\Delta)$) timeslots for a sufficiently long ℓ. Finally, for $\mathcal{T}(\mathcal{P})$, recalling $j = r\ell$, we can write;

$$\begin{aligned}
\tilde{\mathsf{AC}} &= \lim_{j \to \infty} \sup_{\tilde{\mathcal{E}} \in E} \frac{1}{j} \sum_{p \in \tilde{\Pi}} \sum_{i=0}^{j} (|\tilde{T}_{\tilde{\mathcal{E}},p}^{R(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{S(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{R(i)}|) \\
&= \lim_{r \to \infty} \sup_{\tilde{\mathcal{E}} \in \tilde{E}} \frac{1}{j} \sum_{p \in \tilde{\Pi}} \sum_{i=0}^{j} (|\tilde{T}_{\tilde{\mathcal{E}},p}^{R(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{S(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{R(i)}|) \\
&= \lim_{r \to \infty} n^2 C + \frac{2}{r} \sum_{i=1}^{r} (1+\epsilon)\mathsf{AC} = O(\mathsf{AC}),
\end{aligned}$$

as $\mathsf{AC} = \Omega(n^2)$.

Finally, for $\tilde{\mathsf{PC}}$, we note that

$$\begin{aligned}
\tilde{\mathsf{PC}} &= \sup_{\tilde{\mathcal{E}} \in E} \sup_{i \in [0,\infty), p \in \Pi \setminus \mathcal{F}} (|\tilde{T}_{\tilde{\mathcal{E}},p}^{R(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{S(i)}| + |\tilde{M}_{\tilde{\mathcal{E}},p}^{R(i)}|) \\
&\leq \sup_{\mathcal{E}_r \in E, r=1,\ldots} \sup_{i \in [0, \ell+\Delta], p \in \Pi \setminus \mathcal{F}} (|T_{\mathcal{E}_r,p}^{R(i)}| + |M_{\mathcal{E}_r,p}^{S(i)}| + |M_{\mathcal{E}_r,p}^{R(i)}| + n^2 C) \\
&\leq n^2 C + \mathsf{PC} = O(\mathsf{PC}),
\end{aligned}$$

since $\mathsf{PC} = \Omega(n^2)$. ◀

D Additional Related Work

To the best of our knowledge, no earlier work describes a closed-box transformation from permissioned to PoS consensus, that preserves and provides the range of desirable properties

studied in this paper.

PoS blockchain protocols. The first traces [11, 12, 83, 85, 84, 66, 54] of the idea of PoS blockchains can be found in the Bitcoin community [68]. Today, Ethereum [42, 20, 21] is the largest PoS blockchain by market capitalization. The first protocols with a rigorous security analysis included SnowWhite [33] and Ouroboros Praos [35], PoS variants of Sleepy [74] and Ouroboros [53], respectively. Sleepy and Ouroboros are adaptations of Nakamoto's longest-chain protocol [68, 45] to the classical permissioned setting. They replace Nakamoto's proof-of-work with a lottery based on pseudo-random functions. The constructions to handle stake shift in SnowWhite and Ouroboros Praos are not closed-box, but "fold" reconfiguration into the protocol in an inductive way such that the overall structure of Nakamoto consensus with a single block-tree per execution is preserved. To this end, time is divided into epochs, and, conceptually, the stabilized prefix of the longest-chain as of the end of the current epoch governs the rights to block production in the epoch thereafter. Other early PoS protocol include Algorand [26, 46] and Tendermint [58, 17, 18, 32], which implement state-machine replication multi-shot consensus through repeated single-shot Byzantine agreement, without pipelining. A consequence is that these protocols don't show forking, and the stake distribution decided by one single-shot instance can immediately govern the next single-shot instance with ease.

ByzCoin [55] and Hybrid Consensus [73] use proof-of-work to sample a committee among the miners that then run a permissioned consensus protocol with low confirmation latency. Specifically, Hybrid Consensus proceeds in epochs (called "days"), each of which has an associated permissioned consensus instance, with epoch transition and output log construction similar to that of our transformation. Since in Hybrid Consensus an epoch's committee is determined external to the permissioned/PoS protocol, rather than based on the consensus of the epoch prior, consensus instances can overlap around the epoch boundary. Thunderella [75] builds on the techniques of Hybrid consensus to equip high-latency Nakamoto-style consensus with an optimistically responsive fast path. Lewis–Pye and Roughgarden [64] and Budish et al. [19] provide an epoch-based transformation, similar to ours, from permissioned to PoS for HotStuff and Tendermint, respectively. PaLa [25] is a partially synchronous PBFT-style [24] consensus protocol with a builtin reconfiguration mechanism where during reconfiguration both the outgoing and the incoming processes vote for newly produced blocks. Sui Lutris [15] features a hybrid architecture integrating a partially-ordering with a totally-ordering consensus mechanism [78, 48, 10], and provides an epoch-based joint reconfiguration mechanism where the partially-ordering component is effectively paused during epoch transition.

Reconfiguration for state-machine replication. There is a substantial line of work on reconfiguration of replicated state-machines in the traditional distributed-computing literature. Lamport already touched upon reconfiguration of replicated state-machines in the Paxos paper [59] and refined the treatment in subsequent works [61, 62, 60]. The two-mode technique of [61] was subsequently extended to Byzantine faults in [1]. BFT-SMaRt [13] is among the first publicly available libraries that implement a BFT protocol with support for reconfiguration. The popular Raft algorithm [72] also provides a built-in reconfiguration mechanism, but only in a crash-fault model. BChain [39] describes a reconfiguration mechanism that is needed to remove faulty processes and restore liveness. Duan and Zhang [40] provide a recent formal treatment of Byzantine state-machine replication with reconfiguration. Specifically, they provide Dyno, a family of total-order broadcast protocols with reconfiguration carefully woven in a bespoke manner. Tang et al. [82] build on that work.

Group membership & view synchronous communication. The following two classical

distributed computing primitives accommodate dynamic changes in participation: group membership and view synchronous communication [14, 22, 27]. These two primitives were first introduced by Birman and Joseph [14]. Group membership facilitates synchronization among processes regarding the current participants in the system. Specifically, processes advance through an agreed-upon sequence of views, with each view representing the system's membership at a given (logical) time. View synchronous communication [76, 6, 7, 67] combines the group membership primitive with reliable broadcast [16]: guarantees of the reliable broadcast primitive are ensured within a view.

Deterministic reconfiguration in asynchrony. An intriguing and challenging research direction involves the development of deterministic reconfiguration protocols in a fully asynchronous setting. The difficulty of this problem arises from the fact that such protocols cannot depend on consensus, as the seminal FLP impossibility result [43] demonstrates that consensus cannot be deterministically achieved in asynchronous systems. Consequently, agreement on the system's membership can never be guaranteed: reconfiguration must be achieved despite the fact that processes might have different views of the current system membership. This problem was initially explored in the crash failure model [2, 3, 50, 56, 44, 81]. Subsequently, reconfigurable Byzantine-resilient protocols were developed for various applications, including reliable broadcast [47], payment systems [23], and more general purposes [57].